Homework Solutions for Sec. 1.6, 2.1, 2.2

Assigned problems: 1.6: 4, 6, 12, 15, 20, 22, 26, 34, 37, 41, 48, 50, 59.
2.1: 2, 5.
2.2: 6, 7, 9, 14, 16, 17, 24, 32.

Graded problems circled.
Grading scheme: 3 points for each of 5 graded problems, plus 10 points for completing the rest. Take off 0.5 pt. for each missing (ungraded) problem.

Section 1.6: Exercise 26

Find the inverse of $y = \dfrac{e^x}{1+2e^x}$

switch roles of x, y first (optional): $x = \dfrac{e^y}{1+2e^y}$

Solve for y: $x(1+2e^y) = e^y \Rightarrow x + 2xe^y = e^y \Rightarrow x = e^y(1-2x)$

Thus, we have $e^y = \dfrac{x}{1-2x} \Rightarrow \boxed{y = \ln\left[\dfrac{x}{1-2x}\right]}$

Grade: 0.5 pt = switch roles of x, y - either at the beginning, or at the end.
2 pt = algebraic steps, up to $e^y = \dfrac{x}{1-2x}$
0.5 pt = take ln on both sides and get final result

Sec. 1.6; Ex. 41

Given: $\ln(1+x^2) + \frac{1}{2}\ln x - \ln(\sin x)$. Express as a single log.

$$= \underbrace{\ln(1+x)^2 + \ln x^{1/2}}_{\text{Becomes product}}\underbrace{- \ln(\sin x)}_{\text{becomes quotient}} = \ln\left[\frac{(1+x^2)\cdot x^{1/2}}{\sin x}\right]$$

Answer: $\boxed{\ln\left[\dfrac{\sqrt{x}\,(1+x^2)}{\sin x}\right]}$

Grade: 1 pt = rewrite $\frac{1}{2}\ln x$ as $\ln x^{1/2}$
2 pt = rest of the work, leading up to answer
Steps need not be elaborately detailed, but answer alone not sufficient

Sec. 2.1: Exercise 2

Secant line approx for heart rate at $t = 42$ minutes

(a) Using $t = 36$ to 42: $\dfrac{2948-2530}{42-36} = 69.67$ beats/minute

(b) Using $t = 38$ to 42: $\dfrac{2948-2661}{42-38} = 71.75$ beats/minute

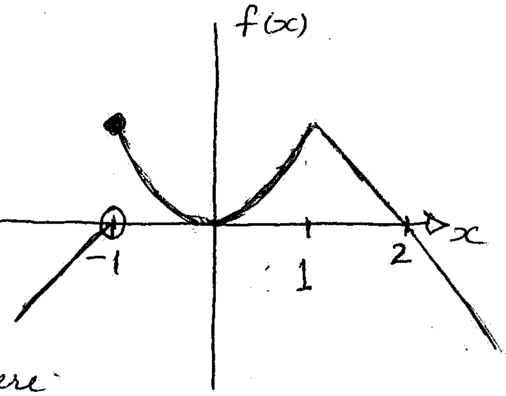
(c) Using $t = 40$ to 42: $\dfrac{2948-2806}{42-40} = 71.0$ beats/minute

(d) Using $t = 42$ to 44: $\dfrac{2948-3080}{42-44} = 66.0$ beats/minute

Grade: 1 point each for (a), (b), (c). Item (d) is not graded.
−1/2 point overall if units missing
−1 point overall, if only answers, with no calculation step.

Section 2.2: Excercise 7

$$f(x) = \begin{cases} 1+x, & \text{if } x < -1 \\ x^2, & \text{if } -1 \le x \le 1 \\ 2-x, & \text{if } x > 1 \end{cases}$$



The limit $\lim_{x \to a} f(x)$ exists for all values of a, except $x = a = -1$, because the left- and right- limits are not equal there.

Grade: 2 points for correct graph, with axes labels and correct open/closed circles.
1 pt = say that limit exists everywhere except at $x = -1$

Sec. 2.2: Excercise 24

$\lim_{x \to 0} \frac{9^x - 5^x}{x}$. Use table of values to estimate limit

The value of the limit appears to be about $\boxed{0.5878}$

	x	$f(x)$
Right side	0.1	0.7111
	0.04	0.6343
	0.002	0.59003
	0.0009	0.58898
	0.00003	0.5878
Left side	-0.2	0.4019
	-0.006	0.5811
	-0.0003	0.5875
	-0.00005	0.5877

Grade:
1 pt = reasonable table of values for $x > 0$
1 pt = reasonable table of values for $x < 0$
1 pt = put together and get reasonable answer