Homework Solutions for Sec. 1.5

Assigned problems: 1.5: 1, 2, 9, 13, 17, 22, 23, 24, 30, 35

Graded problems circled.

Grading scheme: 3 points for each of 5 graded problems $= 3 \times 5 = 15$
10 points for completion of the rest. → Take-off 2 points for missing problems
Total points $= 25$

Exercise 2

(a) $8^{4/3} = (\sqrt[3]{8})^4 = 2^4 = \boxed{16}$

(b) $x(3x^2)^3 = x(3^3x^6) = \boxed{27x^7}$

Grade: 1.5 pt each for (a) and (b).
For each: 0.5 pt = correct answer; 1 pt = some taken steps leading to answer

Exercise 17

(a) Shift 2 units down: $y = e^x - 2$

(b) Shift 2 units right: $y = e^{x-2}$

(c) Reflect about x-axis: $y = -e^x$

(d) Reflect about y-axis: $y = e^{-x}$

(e) Reflect about both: $y = -e^{-x}$

Grade: 0.5 pt each for (a), (b), (c), (d). 1 pt for (e)
Correct answers sufficient

Exercise 22

Want fn. of the form $f(x) = c \cdot a^x$.

We know 2 points on it: $(-1, 3)$ and $(1, \frac{4}{3})$

Plug in and solve for c, a: $\left.\begin{array}{l} 3 = c \cdot a^{-1} \\ \frac{4}{3} = c \cdot a^1 \end{array}\right\}$ Divide lower by upper $\frac{4}{3} \div 3 = \frac{c \cdot a}{c \cdot a^{-1}} \Rightarrow \frac{4}{9} = a^2$

$\therefore a = \pm\sqrt{\frac{4}{9}}$. Since a is the base of an exponential fn, it cannot be negative. Thus, $\boxed{a = 2/3}$

Plug into 2nd eqn. and solve for c: $\frac{4}{3} = c\left(\frac{2}{3}\right)^1 \Rightarrow c = \frac{4}{2} = 2$

Answer: $\boxed{f(x) = 2\left(\frac{2}{3}\right)^x}$

Grade: 1 pt = correctly plug in and get the 2 eqns: $3 = c \cdot a^{-1}$, $\frac{4}{3} = c \cdot a^1$
1 pt = correctly solve for a; 1 pt = correctly solve for c.

Exercise 24

Case I: \$1 million at the end of the month is total compensation

Case II: Total compensation is (in cents): $1 + 2 + 4 + 8 + \cdots + 2^{30-1}$

Let's look at just the last term to see how it compares to \$1 million

2^{29} cents $= 536{,}870{,}912$ cents $= \$5{,}368{,}709.12$ = More than 5 million dollars

Thus, the 2nd method of payment is much better than the 1st

Grade: 1.5 pt = figure out the correct value of payment on the last day in cents; 1 pt = convert it to dollars; 0.5 pt = state answer

Exercise 35

The given function is $f(x) = \frac{1-e^{1/x}}{1+e^{1/x}}$

To prove that it is odd, we must show: $f(-x) = -f(x)$.

Plug in $-x$ and see what happens:

$$f(-x) = \frac{1-e^{-1/x}}{1+e^{-1/x}} = \frac{\cancel{e^{-1/x}}[e^{1/x}-1]}{\cancel{e^{-1/x}}[e^{1/x}+1]} = \frac{e^{1/x}-1}{e^{1/x}+1}$$

$$\therefore f(-x) = \frac{-1+e^{1/x}}{1+e^{1/x}} = -f(x)$$ from the above defn. of $f(x)$

Hence $f(x)$ is odd.

Grade: 1 pt = plug in $-x$ and attempt to see what happens

1 pt = correctly simplify to $f(-x) = \frac{e^{1/x}-1}{e^{1/x}+1}$

1 pt = show/argue that $f(-x) = -f(x)$