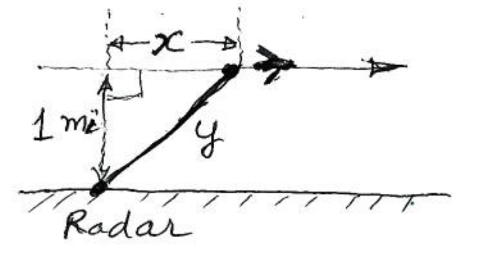
Homework Solutions for Sec. 4.1–4.3

Assigned problems: Sec. 4.1: 3, 5, 13, 23, 29.
Sec. 4.2: 3, 8, 10, 13, 25, 28, 29, 32, 37, 43, 48, 60b.
Sec. 4.3: 2, 5, 6, 9, 12, 20, 22, 33, 41.

Graded exercises circled.
Grading scheme: 3 points each for 5 graded problems, plus 10 points for completing the rest. Take off 0.5 pt. for any missing problem.

Sec. 4.1: Exercise 13

The sketch shows a schematic of the given information.



From the sketch: $x^2 + 1^2 = y^2$

Differentiate w.r. to t: $2x\frac{dx}{dt} + 0 = 2y\frac{dy}{dt}$

We want $\frac{dy}{dt}$ when $x = 2$ mi.

$\frac{dx}{dt}$ = plane's speed = 500 mi/hr. When $x = 2$, $y = \sqrt{1^2+2^2} = \sqrt{5}$ mi

$\therefore \frac{dy}{dt} = \frac{2 \times 500}{\sqrt{5}} = \frac{1000}{\sqrt{5}}$ mi/hr.

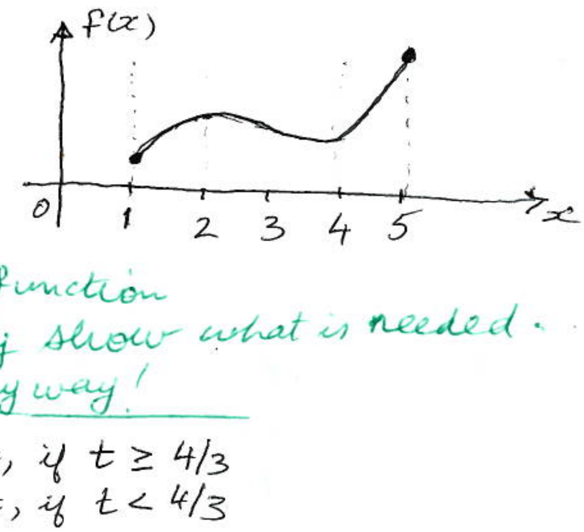
Answer: Distance between plane & radar station is increasing at $\frac{1000}{\sqrt{5}}\frac{mi}{hr}$

Grade: 0.5 pt = using sketch, or otherwise, introduce 2 variables x, y and get the relation $x^2+1^2=y^2$
1 pt = correctly differentiate it with respect to t.
1 pt = figure out values of x, $\frac{dx}{dt}$, y and plug in
0.5 pt = solve and get final answer

Sec. 4.2: Exercise 8

Want an example of a fn. with abs. minimum at 1, abs. maximum at 5, local minimum at 4, local maximum at 2

Ans: There are ∞ correct answers.
One example is shown in the sketch.



Grade:
0.5 pt each for satisfying the 4 stated requirements with a valid function
0.5 pt for axis labels that reasonably show what is needed.
0.5 pt for attempting the problem in any way!

Sec. 4.2: Exercise 28

$$g(t) = |3t-4| \implies g(t) = \begin{cases} 3t-4, & \text{if } t \geq 4/3 \\ 4-3t, & \text{if } t < 4/3 \end{cases}$$

$$\therefore g'(t) = \begin{cases} 3, & \text{if } t > \frac{4}{3} \\ -3, & \text{if } t < \frac{4}{3} \end{cases} \quad \text{and } g'\left(\tfrac{4}{3}\right) = DNE$$

Answer: There is only one critical point. It is at $t = \frac{4}{3}$

Grade: 1 pt = correctly write $g(t)$ in piecewise form
(−0.5 pt if domains reversed, or if they are missing)
1 pt = correct derivative of $g(t)$ at points other than $t = 4/3$
1 pt = find/say $g'(4/3) = \text{DNE}$, making $t = 4/3$ a critical point

<u>Sec. 4.2: Exercise 43</u>

$f(x) = 2x^3 - 3x^2 - 12x + 1, \quad [-2, 3]$

To find abs. extremes, find C.P.'s and plug them into f together with end-points.

$$f'(x) = 6x^2 - 6x - 12 \Rightarrow f'(x) = 6(x^2 - x - 2)$$

$$f' = 0 \Rightarrow x^2 - x - 2 = 0 \Rightarrow (x+1)(x-2) = 0 \Rightarrow \boxed{x = -1,\ x = 2}$$

x	-2	-1	2	3
$f(x)$	-3	8	-19	-8

Abs. minimum: $f(2) = -19$
Abs. maximum $f(-1) = 8$

Grade: 1 pt = take derivative & find correct 2 critical points
1 pt = plug in the right values and evaluate $f(x)$ at these points
1 pt = state what the abs minimum & maximum values are

<u>Sec. 4.3: Exercise 41</u>

Given the following derivative of some function f:

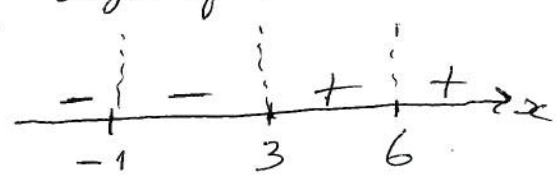
$$f'(x) = (x+1)^2 (x-3)^5 (x-6)^4$$

want to determine intervals on which f is ↑ and where ↓.

Set $f'(x) = 0$ and solve for critical points.

Here the C.P.s are: $x = -1, 3, 6$.

Sign of f'



For the purpose of sign analysis of f', we can ignore the even powers, and simplify it to $f'(x) \approx (x-3)$

Thus $f' < 0$ when $x < 3$, $f' > 0$ when $x > 3$.

Answer: f is decreasing on $(-\infty, 3)$, increasing on $(3, \infty)$

Grade: 1 pt = get correct 3 critical points
1 pt = show some reasonable form of sign analysis of f'
1 pt = correct answer for intervals of increase & decrease