Homework Solutions for Sec. 3.7, 3.8, 3.9

Assigned exercises: Sec 3.7: 3, 4, 9, 11, 14, 15, 18, 22, 28, 29, 38, 40, 44.
Sec. 3.8: 15, 28
Sec. 3.9: 8, 16, 23, 33, 35.

Graded problems circled.

Grading scheme: 3 points for each graded excercise, plus 10 points for completing the rest. Take off 0.5 pt. for any missing exercise.

Sec. 3.7: Exercise 9

$$f(x) = \sin(x) \cdot \ln(5x)$$

Use product rule, plus chain rule inside:

$$f'(x) = [\sin(x)]' \cdot \ln(5x) + \sin(x) \cdot [\ln(5x)]'$$

$$= \cos(x) \cdot \ln(5x) + \sin(x) \cdot \left[\frac{1}{5x}\right] \cdot (5x)'$$

$$\therefore f'(x) = \boxed{\cos(x) \cdot \ln(5x) + \frac{\sin(x)}{x}} \quad \text{OR} \quad \frac{x \cdot \cos(x) \cdot \ln(5x) + \sin(x)}{x}$$

Grade: 1 pt = correctly setup in product rule formula
0.5 pt = get correct derivative of $\sin(x)$
1 pt = correct derivative of $\ln(5x)$
0.5 pt = clean up & get reasonable final answer

Sec. 3.7: Exercise 18

$$y = [\ln(1+e^x)]^2$$

$$\frac{dy}{dx} = 2\ln(1+e^x) \cdot \frac{d}{dx}[\ln(1+e^x)] = 2 \cdot \ln(1+e^x) \cdot \left[\frac{1}{1+e^x}\right] \cdot \frac{d}{dx}(1+e^x)$$

$$= 2 \cdot \ln(1+e^x) \cdot \left[\frac{1}{1+e^x}\right] \cdot e^x$$

$$\therefore \boxed{\frac{dy}{dx} = \frac{2e^x \ln(1+e^x)}{1+e^x}}$$

Grade: Partial credit as follows for each derivative in chain(s)
0.5 pt = get $2\ln(1+e^x)$; 1 pt = get $\frac{1}{1+e^x}$; 1 pt = get $(1+e^x)' = e^x$
0.5 pt = get final answer in some reasonable form

Sec. 3.7: Exercise 38

$$y = x^{\cos x}$$

Apply ln to both sides: $\ln y = \ln[x^{\cos x}] = \cos(x) \cdot \ln(x)$

Differentiate w.r.to x: $\frac{1}{y}\frac{dy}{dx} = \frac{d}{dx}[\cos(x)] \cdot \ln(x) + \cos(x) \cdot \frac{d}{dx}\ln(x)$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = -\sin(x) \cdot \ln(x) + \cos(x) \cdot \frac{1}{x}$$

$$\therefore \frac{dy}{dx} = y \cdot \left[-\sin(x) \cdot \ln(x) + \frac{\cos(x)}{x}\right]$$

$$\Rightarrow \boxed{\frac{dy}{dx} = x^{\cos x}\left[\frac{\cos(x)}{x} - \sin(x) \cdot \ln(x)\right]}$$

Grade: See other side.

Grade: 0.5 pt = apply ln correctly and get $\ln y = \cos x \cdot \ln x$
0.5 pt = correctly differentiate left side & get $\frac{1}{y}y'$
1 pt = correctly differentiate right side
1 pt = multiply by y and replace it by $x^{\cos x}$

Sec. 3.8: Exercise 28 $f = \frac{1}{2L}\sqrt{\frac{T}{\rho}}$, L = Length, T = Tension, ρ = density

(a) (i) $\frac{df}{dL} \longrightarrow$ Write as $f = \frac{1}{2}\sqrt{\frac{T}{\rho}} \cdot L^{-1}$. Then $\frac{df}{dL} = \frac{1}{2}\sqrt{\frac{T}{\rho}}\left(-L^{-2}\right)$

$$\therefore \boxed{\frac{df}{dL} = -\frac{1}{2L^2}\sqrt{\frac{T}{\rho}}}$$

(ii) For $\frac{df}{dT}$, write as $f = \frac{1}{2L\sqrt{\rho}} \cdot T^{1/2} \Rightarrow \frac{df}{dt} = \frac{1}{2L\sqrt{\rho}}\left[\frac{1}{2}T^{-1/2}\right]$

$$\therefore \boxed{\frac{df}{dT} = \frac{1}{4L\sqrt{T\rho}}}$$

(iii) For $\frac{df}{d\rho}$, write as $f = \frac{\sqrt{T}}{2L} \cdot \rho^{-1/2} \Rightarrow \frac{df}{d\rho} = \frac{\sqrt{T}}{2L}\left[-\frac{1}{2}\rho^{-3/2}\right]$

$$\therefore \boxed{\frac{df}{d\rho} = -\frac{\sqrt{T}}{4L\ \rho^{3/2}}}$$

(b) Assume all quantities L, T, ρ are positive.

(i) From (a), $\frac{df}{dL}$ is negative. So, when L decreases, the pitch increases

(ii) Since $\frac{df}{dT} > 0$, when T increases, pitch increases

(iii) Since $\frac{df}{d\rho} < 0$, when ρ increases, pitch decreases.

Grade: 3 points (a), and (b) is not graded.
For (a): 1 point for each of 3 parts. 50/50 split between step(s) & answer

Sec. 3.9: Exercise 33: Blood volume flow rate is modeled by

$F = KR^4$, K = constant, R = radius of blood vessel

To find change in F, we can use differentials or linear approximations.

$$dF = 4KR^3 dR \quad \text{OR} \quad \underbrace{\Delta F}_{\text{change in } F} = 4KR^3 \underbrace{\Delta R}_{\text{change in } R}$$

Relative change requires dividing by original quantity.

$$\frac{dF}{F} = \frac{4KR^3 dR}{KR^4} = 4\frac{dR}{R} \Rightarrow \boxed{\frac{dF}{F} = 4\frac{dR}{R}}$$

Relative change in F is 4 times relative change in R.

* A 5% increase in radius should lead to a $\boxed{4 \times 5 = 20\%}$ increase in blood flow rate.

Grade: 1 pt = get correct expression for dF or ΔF
1 pt = correct expression for $\frac{dF}{F}$ or $\frac{\Delta F}{F}$
1 pt = correctly conclude 20% increase in flow rate.