In Honor of the Nobel Laureates
Robert C. Merton and Myron S. Scholes: A Partial Differential Equation
That Changed the World

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The formal press release from the Royal Swedish Academy of Sciences announcing the 1997 Nobel Prize in economics states that the honor was given “for a new method to determine the value of derivatives. Robert C. Merton and Myron S. Scholes have, in collaboration with the late Fischer Black, developed a pioneering formula for the valuation of stock options. Their methodology has paved the way for economic valuations in many areas. It has also generated new types of financial instruments and facilitated more efficient risk management in society.”

This statement, although true, is just the proverbial tip of the iceberg. The impact of the “pioneering formula” developed by Robert C. Merton, Myron S. Scholes and the late Fischer Black, commonly known as the Black-Merton-Scholes model, is greater than most economists realize.1 In economics, their work on

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1 This paper focuses only on Robert C. Merton’s and Myron Scholes’s research contribution in the area of option pricing. However, Robert Merton has also made seminal contributions to dynamic consumption and investment theory; for an extensive overview, see Merton’s (1992) book on continuous-time finance, or see Merton (1969, 1970, 1971, 1973b). Myron Scholes has also made seminal contributions with respect to estimation issues involving the capital asset pricing model, including issues with respect to the impact of dividends and corporate dividend policy, and taxes (Miller and Scholes, 1978, 1982; Black, Jensen and Scholes, 1972; Black and Scholes, 1974; Scholes and Williams, 1977; Scholes and Wolfson, 1992). For a tribute in these other areas, see Duffie (1998) and Schaefer (1998).
option pricing has not only provided a technique for valuation, but it has also created a new field within finance, known as derivatives, and offered a new perspective on related areas including corporate finance, capital budgeting, and financial markets and institutions. In mathematics and computer science, the direction of study in probability theory and numerical methods has been influenced by problems arising from the use of the option pricing technology. In private industry, the Black-Merton-Scholes option pricing theory has generated not just "new types of financial instruments," but also new organizational structures within firms to help manage risk. Even more dramatically, it has enabled a generation of new industries and new markets.

To understand the broad sweep and impact of the Black-Merton-Scholes option pricing theory—that is, to see the entire iceberg rather than just the tip—one needs to step back and start at the beginning. This is the intention of this essay. As such, this essay discusses the state of the art before their discovery, the discovery itself, and finally the ramifications of the discovery.

A Primer on Options

To understand the Black-Merton-Scholes model, one first needs to understand what an option contract is. To understand the various kinds of options, it helps to begin with the idea of a forward contract.

A forward contract is a financial security that obligates its holder—called the "long"—to purchase an underlying asset for a fixed price at a fixed future date. The fixed future price is called the "forward price," and the fixed future date is called the "maturity" date. The counterparty on the opposite side of the forward contract is called the "short." The short has the reciprocal obligation to sell the underlying asset on the maturity date at the agreed-upon forward price. At the time the forward contract is initiated, no cash changes hands. Indeed, no cash flow occurs from entering into a forward contract until the maturity date, when the underlying asset is exchanged for the forward price. A practical example of a forward contract familiar to many economists is the use of such contracts by farmers to hedge future price uncertainties at the time of harvesting by locking in their commodity's selling price near the beginning of a growth cycle.

There are four basic types of option contracts: European calls, European puts, American calls and American puts. Perhaps surprisingly, the option prefixes have nothing to do with geographical considerations. For concreteness and simplicity, I will concentrate on discussing these option contracts in terms of stocks.

A European call option gives its holder the right, but not the obligation, to purchase a stock at a fixed price—called the "strike" or "exercise" price—at a fixed future date—called "maturity" or "expiration" date. The key difference between the European call option and a forward contract is that for the option contract, the long does not have to buy the stock at the maturity date. For the forward contract, the long
must purchase. A rational holder will, therefore, only exercise the option to purchase at the maturity date if the stock price at that time exceeds the exercise price.

A European put option differs from a European call option in that it gives the right to sell, rather than the right to buy, the underlying stock. An American call option differs from a European call option in that it gives the right to buy at any time after entering the contract and up until and including the maturity date. The European option can only be exercised at the maturity date. Finally, the American put option is identical to the American call except it grants the right to sell.

The key problem in option pricing theory is to determine the value of the option before the maturity date. This calculation determines the price at which the option is bought or sold in the market, and for American-style options, whether the option should be exercised before maturity.

**History of the Black-Merton-Scholes Model**

The origin of modern option pricing theory is Bachelier's (1900) dissertation on the theory of speculation. In this dissertation, Bachelier derived an option pricing formula that today can be recognized as a close kin to the Black-Merton-Scholes formula. In addition, he also developed some necessary mathematics related to diffusion processes and Brownian motions. This dissertation, despite the mathematics and economics being slightly flawed, motivated both Samuelson's (1965) paper on option pricing and Ito's (1951 [1987]) work on stochastic processes.²

Both of these investigations, without the knowledge of their common lineage, separately influenced the development of the Nobel prize winning work. Ito's investigations generated the fundamental theorem of stochastic calculus, known as Ito's lemma, which provides an essential step in the derivation of the Black-Merton-Scholes formula. In addition, as discussed below, Samuelson's work was the precipitating source for Merton's studies on option valuation and for the initial derivation by Black and Scholes of the option pricing formula.

After Bachelier, option pricing theory laid dormant in the economics literature for over half a century until renewed study at MIT by Sprenkle (1961), Samuelson (1965) and Merton and Samuelson (1969). These papers determined an option's price using the maximizing conditions obtained from an investor's optimal portfolio position. As such, the valuation formulas obtained depended on the expected return on the stock or equivalently, the stock's risk premium. This dependency made these formulas difficult to estimate and to use.

The first difficulty is that risk premia shift according to changing tastes and

² Bachelier's influence on Ito's work was acknowledged by Kiyoshi Ito in a private conversation with Robert C. Merton at the Norbert Wiener centenary celebration held at MIT in 1994. In fact, Ito stated that Bachelier's thesis was a far more important influence on him than was Wiener's work.
changing economic fundamentals in an economy, which makes modeling the risk premia and its estimation problematic. At present, there is no generally accepted empirical model for an asset’s risk premium that is consistent with the known regularities present in the data (Connor and Korajczyk, 1995; Ferson, 1995). The second difficulty, perhaps even more important, was that the valuation formulas offered no sense of how to hedge an option using a portfolio of the underlying stock and riskless borrowing or lending. The idea of how to hedge an option is arguably the single most important insight underlying the Black-Merton-Scholes approach. Indeed, it is this insight which enabled the development of new option markets and the formulation of new financial contracts.

The story of how the Black-Merton-Scholes option pricing framework was developed is well-told in Black (1989) and Bernstein (1992). These references are also consistent with first-hand stories related to me by Fischer Black and Robert Merton. Apparently, Fischer Black started working on this problem by himself in the late 1960s. His idea was to apply the capital asset pricing model of Sharpe (1964) and Lintner (1965) to value the option in a continuous time setting. Using this idea, he obtained an implicit solution for the option’s value characterized as a partial differential equation, subject to boundary conditions. However, he could not find its solution. He then teamed up with Myron Scholes, who was also an expert on the capital asset pricing model and who had been thinking about similar problems, to help find a solution. Together, they solved the partial differential equation, using a combination of economic intuition and the earlier pricing formulas.

At this time, Myron Scholes was at MIT. So was Robert C. Merton, who was also at this time applying his ample mathematical skills to various problems in finance, including portfolio theory and option valuation. After various discussions, Merton showed Black and Scholes how to derive their partial differential equation differently; incidentally, the Black-Scholes option pricing formula was first labeled as such by Merton (1970). Merton’s derivation used only an argument based on the continuous-time construction of a perfectly hedged portfolio involving the stock and the call option, and the notion that no arbitrage opportunities exist. The Black-Merton-Scholes option pricing technology had been born.

The Intuition of the Black-Merton-Scholes Pricing Argument

The original hedging argument underlying the Black-Merton-Scholes option pricing technology, although profound in implications, is quite intuitive. The idea contains three parts.

Part one recognizes that a call option on the stock increases in value when the
stock price rises. This is because as the stock price rises, the stock price is more likely to lie above the strike price at maturity. Hence, the call option is more valuable today. Formally, one can write the call value as a function of the underlying stock price (and, of course, time).

Part two uses an implication of this insight. Part one implies that a short position in the stock can be used to hedge against changes in the value of a call option to its holder. A short position in a stock is equivalent to holding a negative number of shares. In practice, the short seller borrows the stock from a broker and sells it to a third party, promising to repay the stock to the broker at a future time, whatever the price of the stock is at that time. To see how this option hedging works, suppose that the stock price rises over a short period, increasing the option's value. The short position in the stock, however, decreases in value since repaying the stock to the broker in the future will cost more. These changes in value partially offset each other, hence, a partial hedge. Conversely, if the stock price falls over a short time period, the value of a call option for that stock decreases in value, but the short position in the stock increases in value. Again, this yields a partial hedge.

Part three modifies the partially hedged position in the long call option and short stock to make it an exact hedge over a short time period. Indeed, under certain hypotheses, it is possible to determine the exact number of shares of stock (less than one) to short for each long call option, so that for any change in the stock price, the change in the value of the call option is exactly offset by the change in value of the short position in the stock. This gives a perfectly hedged portfolio. This perfectly hedged portfolio would then uniquely identify the option's arbitrage free price. Why? The portfolio requires a known initial investment and it is riskless over a given time period. Hence, to avoid arbitrage (profits at no risk of a loss), because the hedged portfolio is riskless, it must earn the riskless rate, which is observable. Otherwise, profits could be made by buying the hedged portfolio and selling the riskless asset, or vice versa. This restriction determines the change in the value of the call option as a function of the value of the underlying stock price and the riskless rate. More precisely, it determines a partial differential equation satisfied by the call's value, whose solution is the Black-Merton-Scholes formula. (The partial differential equation and its solution are given in the appendix.)

The idea of constructing a perfectly hedged portfolio is the key insight of the Black-Merton-Scholes approach, more important than the valuation formula itself. Indeed, if one considers the meaning of a perfectly hedged portfolio, it becomes apparent that it implies that a position in the stock and the riskless asset can be created that exactly duplicates the changes in value to the call option. This position is called a "synthetic option" because it generates the call option's payoffs without explicitly taking a position in the option itself. We will return to the usefulness of synthetic call options later in this essay.

The solution to the partial differential equation obtained from the perfectly hedged portfolio, the Black-Merton-Scholes formula, depends on the option's strike price, its maturity date, the current date, the current stock price, the risk-free
interest rate, and the stock price’s volatility (as measured by its standard deviation) per unit time. All of these quantities are either directly observable or easily estimated. Perhaps more important is what the call option’s value does not depend on. The Black-Merton-Scholes formula does not explicitly depend on the expected return on the stock or, equivalently, the stock’s risk premium. Recall that it was this dependency that made the option price formulations of the early and mid-1960s unuseable in practice. It does not depend on the stock’s expected return because the perfectly hedged portfolio argument is valid regardless of the change in the stock price or the probability of its changing.

Embedded within the derivation of the Black-Merton-Scholes formula are two key assumptions. The first is that the risk-free interest rate is constant. The second is that the stock price’s distribution has a constant volatility. Both assumptions are simplifications of reality, more reasonable for short-term (a year or less) options on assets whose returns are uncorrelated with changes in interest rates, than for long-term options or options on assets whose returns are correlated with interest rates. Typically, these conditions hold true for short-term options on equities or equity indices. These conditions do not apply very well to interest rate derivatives or foreign currency swaps.

The generalization of these two assumptions represents an important component in the evolution of option pricing theory. The first step in the process of generalizing these assumptions began with Robert Merton’s (1973a) original publication. In this paper he extended the Black-Merton-Scholes model to the case where the volatility depends on the stock price (footnote 57) and where there is one risky zero-coupon bond trading whose maturity matches the maturity of the option. The original Black-Scholes formula only applied to European call and put options. Merton’s (1973a) paper also showed how to apply this technology to arbitrary derivatives on the stock, including an American call and put option. Later, Merton (1976a) also generalized the model to include jumps in stock prices. These insights paved the way for the application of this technology to other fields within economics.

The Contribution of the Black-Merton-Scholes Argument to Industry Practice

The Black-Merton-Scholes pricing argument has been central to the development of derivative markets. Options provide a vehicle useful for both speculation and insurance. As an example of the insurance motive, consider an investor who owns a stock that has greatly appreciated in value. Suppose that this investor wants to continue holding the stock, but is nervous about the stock losing value. To insure against this risk, the investor can purchase a put option on the stock with a strike price equal to the stock’s current value. In this case, the put option insures the stock’s value at the current level. As an example of the speculative motive, consider
an investor who is very bullish on a stock, and who wishes to borrow to buy the stock and take advantage of leverage. The investor can implement this speculative strategy by purchasing a call option on the stock—that is, by purchasing the right to buy a certain amount of the stock at a predetermined price at a set time in the future. The initial investment is only the option premium, which is usually a small percentage of the stock's current value.

In thinking about the buying and selling of options, it is important to remember that options trade in a "zero-supply" market, which simply means that for each option sold there must be an option purchased. Options trade in two types of zero-supply markets: organized exchanges, like the Chicago Board Options Exchange, and over the counter (OTC). Exchange-traded options have a physical location or single computer network where all trades take place. In addition, exchange-traded option contracts are standardized in terms of the underlying asset's cash flows, maturity date and strike prices. In contrast, an OTC market consists of multiple brokers and investment bankers, communicating and executing trades via phones and computer screens. OTC contracts are often less standardized and they are tailored to meet the needs of particular traders.

For an exchange-traded option market to grow and succeed, it needs a large investor demand for the derivative financial instrument based on both types of investment motives—insurance and speculation. This is because an exchange needs both buyers and sellers if trade is to occur. In addition, for an exchange-traded option market to succeed, it must be costly for individuals to construct the options synthetically, either in terms of transaction costs or know-how. Otherwise, investors would simply invest in the hedge portfolio outlined by the Black-Merton-Scholes hedging argument, which would make an organized option market redundant and unnecessary. These conditions seem to be satisfied by markets in many underlying securities—including equities, foreign currencies, commodities, government bonds—and in many different countries.

For over the counter markets to flourish, the same argument is sufficient, but not necessary. If the demand for a particular type of option is one-sided—say, because a large group of investors wish to act based on the insurance motive—then a broker in the OTC market can create the other side of the market synthetically. Rather than finding a speculator on the other side of the market, the broker can sell the option and synthetically construct the other side of the market. Again, the presence of transaction costs and know-how are necessary to explain why the services of a middleman or broker are necessary. Moreover, in practice, the portfolio is only approximately hedged, due to market frictions and possible model misspecification. This approximate hedging implies that the middleman or broker is speculating, but in a dramatically reduced and controlled manner. Again, the rapid growth of OTC trading suggests that these conditions are satisfied in many markets in many underlying securities, including swaps on interest rates and foreign currencies, mortgage-backed securities, and credit derivatives.

For both exchange-traded and OTC markets, the Black-Merton-Scholes tech-
nology facilitated trading by allowing investors to understand the risks of derivatives better. In the late 1960s and early 1970s, at the time that the Black-Merton-Scholes discoveries were being formulated, call and put options on equities were considered esoteric instruments, and they traded only in limited quantities between a few brokers and investment banks. In fact, this lack of exchange-traded options partly accounts for the difficulty that Black and Scholes experienced in publishing their seminal paper. The remaining difficulty has been attributed to the novelty and difficulty of the mathematics employed. In April 1973, around the time of the publication of the Black-Scholes model, the Chicago Board Options Exchange began trading the first listed options in the United States.

Since that time, the growth in exchange-traded and over the counter traded options on equities, indices, foreign currencies, commodities, and interest rates has been phenomenal. It is widely believed that the growth in these option markets was enabled by the Black-Merton-Scholes option pricing technology. The option pricing technology provided a tool useful for identifying profit opportunities and thereby generating the necessary market demands. For OTC markets, the Black-Merton-Scholes technology was essential to allowing brokers to synthesize the other side of the market.

In response to these new derivatives markets, new firms were created and new departments in existing firms and banks were formed to take advantage of these new trading opportunities. The Black-Merton-Scholes technology provided the required “know-how” necessary for managing the risks of these new organizations.

The Field of Derivatives

The Black-Merton-Scholes option pricing technology created a new field within finance in the area of asset pricing or investments known as derivatives. It might be argued that the field of derivatives existed before the work of Black, Merton and Scholes, based on the option-pricing papers written in the 1960s, and discussed earlier in the context of the history of the Black-Merton-Scholes formula. Even given this perspective, however, one must concede that the Black-Merton-Scholes technology was the equivalent of applying a miracle-growing fertilizer to a sparse and relatively infertile field within finance.

To underscore that “derivatives” have indeed become a field of study, one need do no more than count the number of academic and professional journals devoted almost exclusively to this area. A partial list includes: Journal of Derivatives, Review of Derivatives Research, Review of Futures Markets, Mathematical Finance, Applied Mathematical Finance, Journal of Risk, Finance and Stochastics, Journal of Financial Engineering, International Journal of Theoretical and Applied Finance, and the Journal of Computational Finance. This does not include the standard journals in finance and economics where papers on derivatives also regularly appear (as in many of the references at the end of this paper).
The development of the field of derivatives has proceeded along three related lines of inquiry: a study of the mathematical foundations of arbitrage pricing theory; generalizations of the assumptions; and empirical and computational implementation. We will talk about each of these developments in turn.

Mathematical Foundations

The study of the mathematical foundations of derivatives pricing commonly revolves around the meaning of the phrases "no-arbitrage" and "complete markets." No-arbitrage means that prices in markets are such that trading opportunities for profits with no risk are nonexistent. A complete market is one where synthetic construction of any security or derivative is possible. Both concepts underlie the Black-Merton-Scholes option pricing argument. The Black-Merton-Scholes option pricing argument was correct, but the mathematical details were not explicitly formulated. To use (and verify) the argument, precise mathematical definitions needed to be given to these concepts.

The "no-arbitrage" and "complete market" concepts were first formalized in papers by Harrison and Kreps (1979) and Harrison and Pliska (1981). These papers showed that, under suitable hypotheses, these concepts could be characterized using the notion of a martingale probability distribution. A martingale is a stochastic process whose expected future value equals its current value. Martingales have had a long history of study in probability theory, so significant mathematical machinery has been made available by this discovery.

Essentially, the concept that no arbitrage possibilities exist is equivalent to the notion that the stock's expected future value is the same as its current price (appropriately discounted), which is to say that the stock is properly priced. Any changes in the stock's price through time are caused by unanticipated and random events. For example, if stock prices follow a "random walk," then stock prices follow a martingale. Essentially, the concept of market completeness is equivalent to the notion that there is only one martingale probability distribution. That is, there is only one value for a derivative security that is consistent with the prices of the underlying assets. This is because the derivative can be synthetically constructed and we know the fair price of the underlying assets used in the construction.

This probabilistic characterization of the economic concepts of no-arbitrage and market completeness introduced mathematicians to the field of derivatives. This, in turn, led to various advances in the underlying probability concepts themselves. This area of investigation is still quite active. In fact, it has its own

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4 This characterization of the stock's price is often described as "risk-neutral valuation" (Cox and Ross, 1976), because under the martingale probability distribution, the current stock price can be written as a discounted expected future value, where the risk-free rate provides the appropriate discounting factor. This is the approach that one would use to compute present values in an economy where all investors are risk neutral with common beliefs represented by the martingale probability distribution. A precursor to this insight can be found in the util-probabilities of Merton and Samuelson (1969) and Merton's (1972) description of the Black-Scholes partial differential equation.
academic journals, for example: *Mathematical Finance, Applied Mathematical Finance,* and *Finance and Stochastics,* and its own international society, founded in 1996—the Bachelier Finance Society. Papers on the mathematical foundations of derivatives now also appear with some regularity in mathematical journals such as the *Annals of Probability, Stochastic Processes and Their Applications, Applied Probability,* and *SIAM Review.*

This interest by mathematicians in derivatives has had some unexpected consequences. One is that various math and engineering departments—for example, at Carnegie Mellon University, Cornell University, the University of Chicago, the University of Michigan, and New York University—have recently introduced masters programs specializing in derivatives and mathematical finance. A second is that mathematicians and physicists can now find alternate and high-paying demand for their skills on Wall Street.

**Generalizations of the Assumptions**

Two key assumptions underlying the Black-Merton-Scholes model, as mentioned earlier, are constant risk-free interest rates and a constant volatility for the underlying asset. The generalizations of these assumptions are by now quite considerable. To some extent, they followed markets conditions and the need for these generalizations by industry practitioners.5

In the 1970s, the application of derivative models to industry practice was focused primarily on equities and foreign currencies. Generalizations of the Black-Merton-Scholes formula included models in which volatility was random, rather than constant, and models in which stock prices jumped, rather than moving smoothly; Jarrow and Rudd (1983) offer a review of the state of the art in the 1970s. These models typically involve only one or two underlying assets and nonrandom interest rates.

In the 1980s, increased interest rate volatility occurred due to double-digit inflation and the shift from fixed to floating currency exchange rates with the collapse of the Bretton Woods agreement. The increased volatility of interest rates created a new demand for interest rate derivatives, for both motives of insurance and speculation. The assumption of constant interest rates needed to be relaxed. This extension was provided by Ho and Lee (1986), Black, Derman and Toy (1990), and in its greatest generality by Heath, Jarrow and Morton (1992). These models, which represented the first significant development in option pricing theory in over a decade, built upon the mathematical foundations of the Black-Merton-Scholes option pricing theory. These were significant because it quickly became apparent

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5 There are other implicit assumptions underlying the Black-Merton-Scholes model that I have not mentioned including competitive and frictionless markets. The relaxation of these assumptions have also been studied, but I do not emphasize them here: the assumption of competitive markets (Jarrow, 1994; Cherian and Jarrow, 1995); assumptions of no taxes (Scholes, 1976); and assumptions of no transaction costs (Karatzas, 1997).
that all future derivatives pricing applications could be handled as straightforward extensions of this technology. This includes foreign currency pricing problems, credit risk pricing problems, and commodity futures pricing problems.

The common characteristic of these "arbitrage-free term structure models," as they are called, is that there is an evolution of multiple term structures of futures prices or interest rates upon which the derivatives are written. If the no-arbitrage condition holds, analogous to the Black-Merton-Scholes formula, then option prices only depend on the term structure's initial values and volatilities. These later derivative applications were those implemented in the 1990s; Jarrow and Turnbull (1996) provide a review of these models.

At the dawn of the 21st century, after more than 25 years of trading experience in ordinary calls and puts, these vanilla options are now thought of as commonplace. Due to increased market awareness of the possibilities of derivatives and increased price volatility in various financial markets, the market now demands trading in more exotic options, options that can partition the cash flows differently and/or are less expensive to trade. For example, European digital options pay a fixed amount if a stock price exceeds the strike price at maturity, while European barrier options cease to exist if certain stock price levels are crossed (Jarrow and Turnbull, 1996, ch. 19–20). New second-generation models have arisen in which ordinary plain-vanilla calls and puts are used to construct synthetic and exotic options (Derman and Kani, 1998).

Empirical and Computational Implementation

The empirical validation of the Black-Merton-Scholes formula and its generalizations has become a significant research endeavor over the past 25 years. The literature is replete with studies investigating the Black-Merton-Scholes formula and its generalizations. For many markets, including equities and foreign currencies, the Black-Merton-Scholes model fits reasonably well, with some predictable biases (Black and Scholes, 1972; Merton, 1976b; Campbell, Lo and MacKinlay, 1997, ch. 9–10). In fact, predictable biases created by assuming constant interest rates or constant volatility partially provided the empirical motivation for the generalizations and extensions of the Black-Merton-Scholes option pricing technology already discussed. An interesting line of investigation that surfaced in this regard involved the notion of an implicit (or implied) volatility.6

An implicit volatility is that value of the stock's volatility that equates the market price of a call to the Black-Merton-Scholes value (given all the other elements like strike price, maturity date, stock price, and so on). The volatility is implicit because it is inferred from market prices rather than estimated as a sample standard deviation using historic stock price observations. Implicit volatilities have

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6 There is an important branch of the literature that studies the information revelation role played by implicit volatilities in options market equilibrium. For a discussion, see Cherian and Jarrow (1998) and references therein.
been studied in great detail with respect to their forecasting ability and behavior (Campbell, Lo and MacKinlay, 1997). The evidence, although mixed, leans towards implicit volatilities being better forecasters of future volatilities than historic volatilities.

Implicit volatilities are a convenient summary measure of an option's value. This is because the option's value is strictly increasing in the stock's volatility, everything else constant. In this regard, they are comparable across options with different strikes, different maturities, even different underlying assets. So intuitive is the notion of an implicit volatility that in many markets, including equities and foreign currencies, traders actually quote option prices using implicit volatilities. Only after a trade is completed are implicit volatilities transformed back into dollar prices for final settlement and delivery. This is yet another way in which the Black-Merton-Scholes option pricing technology has had a tremendous impact on industry behavior and practice.

The application of this technology to derivatives pricing and hedging has also been the impetus for advances in the area of computation. To use the Black-Merton-Scholes pricing technology, one needs to be able to compute option prices for numerous different instruments quickly and repeatedly. In many cases, the computational problems can be substantial, since the expected distribution of future cash flows from the derivative can be very complex. In the case of Asian options, for example, the payoffs depend on an average value of the underlying asset over the option's life, which means that an option's current payoffs depend on all past values of the underlying asset, along with other factors. Due to the complexity of the computations, which often involve Monte Carlo simulations and other methods, much research effort has been devoted to studying the methods for computing these values. This research focus is one of the origins of the term "financial engineering." For current issues in this branch of the literature, see the Journal of Computational Finance.

The Field of Corporate Finance

The Black-Merton-Scholes option pricing perspective has had a profound impact on corporate finance, which is the study of the firm's financial and investment decisions. It has had an especially powerful impact on several related areas of corporate finance: contingent claims analysis, capital structure policy and capital budgeting.

Contingent claims analysis is the application of option pricing theory to the valuation of corporate liabilities like debt, equity and convertible bonds. Black and Scholes (1973) and Merton (1970, 1973a, 1974) planted the seeds for this area of study in their original papers. To understand this application, consider a firm that has only a single issue of debt outstanding. Suppose that the debt issue has no coupon payments and that it has a fixed principal payable at the maturity of the
loan (say, 10 years). When the debt is scheduled to mature, debtholders can either accept the payment of the debt, or if the firm cannot pay the debt, then the debtholders can seize the assets of the equityholders. Black, Merton and Scholes noted that in effect, the firm's equity can thus be considered as a European call option on the firm's assets, with a strike price equal to the face value of the debt and a maturity date equal to the maturity of the debt.

This notion is simple, but powerful. It implies, for example, that the debtholders are actually the ultimate owners of the firm's assets, having written a call option on them to the equityholders. Of course, this contrasts with the traditional finance perspective that the sole owners of the firm are thought to be the equityholders. Contingent claims analysis argues that both the debt- and equityholders "own" the firm; the debtholders get the first stream of payments from the firm's assets and the equityholders get the residual. This notion allows one to understand the conflict between debt- and equityholders and why various corporate liabilities are structured to reduce this conflict.

To illustrate this conflict, consider the simple firm discussed earlier, with only a single issue of debt with no coupons. Let interest rates be constant, and let the firm's asset returns exhibit constant volatility. Since the equity is a European call option on the value of the assets, we know from the Black-Merton-Scholes formula that the value of equity is increasing in the volatility of the firm's asset returns. Essentially, this occurs because greater volatility raises the chance of an extremely good outcome for equityholders, but since the losses of equityholders are bounded at zero, they do not need to worry about whether losses below a certain level are slight or considerable. Consequently, once debt is in place, equityholders have an incentive (acting through management) to increase the risk of the firm's assets, thereby increasing the value of their equity and decreasing the value of the debt. This effect is known as the agency cost of debt (Jensen and Meckling, 1976).

Clearly, maximizing firm value may be distinct from maximizing the equity value. Equityholders and management will tend to choose the latter at the expense of the debtholders. However, rational debtholders will anticipate this difference and price the original debt issue accordingly. This generates the possibility for an optimal debt/equity ratio, providing an alternative to the Modigliani and Miller (1958) irrelevance theorem showing that under certain assumptions, the debt/equity ratio simply involves different ways of slicing up the corporate pie, but is irrelevant to the question of maximizing firm value. This equityholder/debtholder conflict can also be used to understand why, for example, firms issue convertible debt to avoid these agency costs (Brealey and Myers, 1991).

This notion of contingent claims also allows one to quantify the probability that a firm's debt will default, generating a credit rating methodology comparable to that credit rating used by Moody's or Standard & Poor's. This topic is of considerable current interest both in the academic and professional communities (Jarrow, 1998).
Most directly, this option pricing framework allows one to value and hedge corporate liabilities using the option pricing technology. For example, a call provision allows the firm to repurchase a debt issue from the debtholders at some future date, following some predetermined repurchase schedule. Such embedded options exist in most outstanding corporate debt issues and the option pricing technology provides the tools to evaluate them; Mason and Merton (1985) offer a review.

Capital budgeting is the study of the optimal selection of a firm's investment projects. Finance textbooks teach correctly that investment projects should be selected to maximize the net present value of the firm. What option pricing theory adds is the recognition that most investment projects have options embedded within them. Purchasing land, for example, has the embedded option of developing it immediately or at some future date. Further, whatever type of real estate usage is selected at the time of development—farming, residential building, commercial building—typically involves other embedded options. The Black-Merton-Scholes option pricing argument provides a method for valuing these options, which are commonly called "real" options.

Real option pricing theory has been applied to oil leases, forest development, the mining of precious metals, land development, the production process of a firm and the use of excess capacity (Dixit and Pindyck, 1994; Amram and Kulatilaka, 1999). The issues of real option pricing often become entangled in the conflict between debt- and equityholders, since equityholders may wish to use these options to increase the volatility of the firm's returns. Again, the agency costs of debt generate the possibility for an optimal debt/equity ratio and provide a better understanding of the pecking order of corporate debt (Myers and Majluf, 1984).

**Broader Insights Across Economics**

Within economics, the area most influenced by the Black-Merton-Scholes option pricing theory has been money and banking, or more broadly, financial markets and institutions. In turn, insights from these areas influence both monetary policy in macroeconomics and exchange rate determination in international economics.

Within the field of money and banking, option pricing theory has been used to model Federal Deposit Insurance Corporation (FDIC) premiums as the value of a put option, issued by the government to the individual banks (Merton, 1977). This put option protects the depositors' account balances from default by the bank. This insight allows one to understand the incentives of bank management under a FDIC system. In the area of asset and liability management, the Black-Merton-Scholes option pricing technology has been applied to the valuation of mortgage-backed securities, credit card portfolios, certificates of deposits, NOW (Negotiable
Orders of Withdrawals) accounts—in fact, to the entire balance sheet of the bank (Jarrow and van Deventer, 1998).

Most recently, the application of option pricing to corporate loans, the mainstay of commercial banking, has received much attention. Corporate loans often default, and this possibility necessitates higher required interest payments. The difference between corporate interest rates and Treasury (or default-free) rates is known as a credit risk spread. Implicit in this credit risk spread is the probability of default and the recovery rate in the event of default. Because corporate debt has different maturities, there is a term structure of credit risk spreads. A recent approach to pricing corporate debt is to model the bankruptcy process implicitly by modeling the arbitrage-free evolution of the term structure of credit risk spreads to Treasuries (Jarrow and Turnbull, 1995). This approach recovers the implied default probabilities, recovery rates and can even be used to generate credit ratings. Option pricing formulas have become an essential tool for banks and bank regulators in thinking about capital standards and financial positions.7

In other areas of economics, financial instruments like life insurance and pension fund guarantee contracts can be priced and hedged using the Black-Merton-Scholes technology. With the aging population, this area of investigation is growing (Bodie and Merton, 1992). Option valuation theory in the form of real options can also be used in industrial organizations to understand a firm’s exit and entry decision and in labor economics to understand a firm’s temporary versus permanent hiring decisions (Dixit and Pindyck, 1994). It has been used in the area of international economics to understand exchange rate behavior (Baldwin and Krugman, 1989; Dixit, 1989). In general equilibrium theory, the mathematical foundations of arbitrage pricing theory have been used to generate existence proofs for competitive equilibrium in both complete and incomplete markets (Duffie, 1996), to characterize Arrow-Debreu securities in markets with a continuum of states (Breeden and Litzenberger, 1978), and to provide sufficient conditions for complete markets (Ross, 1976). Merton (1998) offers additional applications of option pricing theory to economics.

Conclusion

The insights of Robert C. Merton, Myron Scholes, and Fischer Black on option pricing have had a substantial impact in many areas. In the social

7 Credit risk measurement is an important consideration in the determination of Value-at-Risk (VAR) computations, required by banks, for their capital determination. VAR is a measure of the value that a bank’s portfolio is no smaller than with some pre-stated probability, like 5 percent, over a given time period, usually 7 or 10 days. Although VAR computations are required by regulators, there is significant academic debate on the usefulness of VAR as a risk measure (Artzner, Delbean, Eber and Heath, 1999).
Appendix
Deriving the Black-Merton-Scholes Formula

This derivation does not highlight the mathematical details inherent in the use of stochastic calculus. Rather, it concentrates on the economic intuition underlying the derivation. For the details see Merton (1973a, 1992).

Formally, consider a portfolio consisting of a call option on the stock with strike $K$ and maturity $T$, and $N$ shares of the underlying stock. Let the portfolio be constructed at time $t$ when the stock price is worth $S(t)$ and the call is worth $c(t, S(t))$. At construction, the value of the portfolio is

$$c(t, S(t)) + NS(t).$$

This equals the value of the long call position plus the value of the $N$ shares in the stock.

We choose $N$ so that the change in the value of this portfolio over $[t, t + \Delta t]$ is riskless. The change in the value of this portfolio over $[t, t + \Delta t]$ is

$$\Delta c(t, S(t)) + N\Delta S(t) \quad \text{where} \quad \Delta S(t) = S(t + \Delta t) - S(t)$$

and

$$\Delta c(t, S(t)) = c(t + \Delta t, S(t + \Delta t)) - c(t, S(t)).$$

Note that the change in the value of this portfolio is solely due to stock price changes and not to cash inflows or outflows. In particular, it is assumed that the underlying stock pays no dividends over the option’s life.

Using a stochastic version of a Taylor series expansion, known as Ito’s lemma
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(see Ikeda and Watanabe, 1981, ch 3), one can show that the change in the value of the portfolio is

$$
\Delta e(t, S(t)) + N\Delta S(t) = \frac{\partial e(t, S(t))}{\partial t} \Delta t + \frac{\partial e(t, S(t))}{\partial S(t)} \Delta S(t) + (1/2) \frac{\partial^2 e(t, S(t))}{\partial S(t)^2} \sigma^2 S(t)^2 \Delta t + N\Delta S(t).
$$

In this expression, $\sigma^2 \Delta t$ corresponds to the variance of the return on the stock, $\Delta S(t) / S(t)$, over $[t, t + \Delta t]$.

The random terms in this expression are those involving the change in the stock price $\Delta S(t)$. By choosing

$$
N = -\frac{\partial e(t, S(t))}{\partial S(t)},
$$

the random terms on the right side of the expression cancel, making the change in the value of the portfolio deterministic (riskless). Holding $N$ shares of the stock to each long call creates the perfectly hedged portfolio. The determination of this $N$ was the purpose of this calculation. $N$ is called the option’s delta.

Hence, to avoid arbitrage, the change in the value of the hedged portfolio must be equal to the riskless return on the initial investment, i.e.

$$
\frac{\partial c(t, S(t))}{\partial t} \Delta t + (1/2) \frac{\partial^2 c(t, S(t))}{\partial S(t)^2} \sigma^2 S(t)^2 \Delta t = \left[ c(t, S(t)) - e^{-r\Delta t} \frac{\partial e(t, S(t))}{\partial S(t)} S(t) \right] r\Delta t.
$$

The left side of this expression is the change in the value of the perfectly hedged portfolio. The right side of this expression is the initial investment times the riskless rate $r$ earned for the time period $\Delta t$.

Finally, in this expression the $\Delta t$ term cancels, giving a partial differential equation for $c(t, S(t))$ subject to the boundary condition

$$
c(T, S(T)) = \max[S(T) - K, 0].
$$

The solution of which is the Black-Merton-Scholes formula:

$$
c(t, S(t)) = S(t) \Phi(d_1) - Ke^{-r(T-t)} \Phi(d_2)
$$

where

$$
d_1 = \frac{\log(S(t)/Ke^{-r(T-t)})}{\sigma \sqrt{T-t}} + (1/2)\sigma \sqrt{T-t}, \quad d_2 = d_1 - \sigma \sqrt{T-t},
$$

and $\Phi(\cdot)$ is the cumulative normal distribution function.
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