Chapter 3  Reconsidering Value and Price

What does it mean when someone says that a particular stock is “undervalued” or “overvalued”? When a stock, or any asset for that matter, is listed on an organized exchange, then it will have a market price. Is the market price not an appropriate measure of value? Is there a conceptual difference between the price of an asset and its value? The answers to these questions are not just for academics attempting to understand how asset markets function. How one answers these questions and conceptualizes prices and value will influence their investment strategies. Those that believe there is little point to the value/price distinction will most likely follow passive strategies. On the other hand, those that believe in the importance of the value/price distinction will find an active strategy useful.

The current chapter begins to address the value/price distinction. In doing so, the chapter gives a concise introduction to several of the topics covered in the following chapters. Section 3.1 puts the distinction in a historical context and traces out the competitive process. Section 3.2 introduces a general definition of value with emphasis on the notion of present value. Sections 3.3 and 3.4 applies the general definition of value to bonds (debt) and stocks (equity) respectively. Section 3.5 presents an alternative to the previous sections by introducing ideas where the value/price distinction no longer exists. The issues addressed throughout the chapter are merely introduced. It will be the goal of following chapters to round out our understanding of the issues.

3.1  Adam’s Original Sin

Economists have debated the meaning of value since Adam Smith’s *Wealth of Nations* (1776). Smith had noted two ways in which the term value could be understood. First, the value of a commodity could refer to what you could get in terms of other commodities in exchange (i.e., exchange-value). Second, value could refer to the benefit you obtain from the commodity (i.e., use-value). The first is a social and objective reference, while the second is personal and subjective.

In the 1800s, the debate over the meaning of value took on greater significance. Smith had attempted to find some objective basis for exchange-value. The economist David Ricardo picked up on a suggestion by Smith that the objective basis of exchange-value could be determined by the amount of labor it took to produce the commodity. Ricardo put forth a detailed account of how the labor embodied in a commodity would regulate exchange-value. To use Smith’s example, if a beaver required 1 day to hunt and a deer 2 days, then the exchange-value of a deer should be 2 beavers. If the exchange-value of a deer were only 1 beaver, then deer hunters would be motivated to stop hunting deer and begin hunting beavers – causing the supply of beavers to increase.

1 Smith had dismissed this basis for exchange-value for a capitalist system. Labor embodied in a commodity would only regulate exchange-value in an ‘early and rude’ state of society.
2 It might be of some interest to note that Ricardo was not an academic. In fact, Ricardo made his fortune in the financial markets and retired early to a life of study and ease. It was only in retirement that Ricardo turned his full attention to political economy.
and deer to decrease until the exchange-value established by the labor embodied was established. Ricardo demonstrated how the same process would work in more advanced economic settings (i.e., capitalism).

In Ricardo’s hands, it became clear that value and price, though related, were clearly different concepts. After all, the units of the two were different – value measured in labor time (e.g., hunting days) and prices measured in monetary units. However, the relative value (e.g., 2 beaver for 1 deer) would ultimately regulate the relative prices. Prices of commodities might deviate from their underlying values for all sorts of reason – temporary disruptions in supply, changes in fashion, etc. Analyzing these prices was much less interesting than analyzing what the prices were deviating from. The actual prices observed in the market were termed market prices, while the long-run prices that were at the center of the deviations were termed natural prices. It was the natural price that value tended to regulate.

Smith had made the distinction between market price and natural price. The distinction continues to be – under various names – a center piece of the analysis of how markets function. Ricardo, following Smith closely, provides the most straightforward description of how natural prices serve as the center of gravitation for market prices. It is worth quoting Ricardo at length here.

Let us suppose that all commodities are at their natural price, and consequently that the profits of capital in all employments are exactly at the same rate, or differ only so much as, in the estimation of the parties, is equivalent to any real or fancied advantage which they possess or forego. Suppose now that a change of fashion should increase the demand for silks, and lessen that for woollens; their natural price, the quantity of labour necessary to their production, would continue unaltered, but the market price of silks would rise, and that of woollens would fall; and consequently the profits of the silk manufacturer would be above, whilst those of the woollen manufacturer would be below, the general and adjusted rate of profits. Not only the profits, but the wages of the workmen, would be affected in these employments. This increased demand for silks would however soon be supplied, by the transference of capital and labour from the woollen to the silk manufacture; when the market prices of silks and woollens would again approach their natural prices, and then the usual profits would be obtained by the respective manufacturers of those commodities. It is then the desire, which every capitalist has, of diverting his funds from a less to a more profitable employment, that prevents the market price of commodities from continuing for any length of time either much above, or much below their natural price. It is this competition which so adjusts the exchangeable value of commodities, that after paying the wages for the labour necessary to their production, and all other expenses required to put the capital

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3 Actually, the distinction goes back much further than Smith to the writings of William Petty and Richard Cantillon.
employed in its original state of efficiency, the remaining value or overplus will in each trade be in proportion to the value of the capital employed. In the 7th chap. Of the *Wealth of Nations*, all that concerns this question is most ably treated. Having fully acknowledged the temporary effects which, in particular employments of capital, may be produced on the prices of commodities, as well as on the wages of labour, and the profits of stock, by accidental causes, without influencing the general price of commodities ... we will leave them entirely out of our consideration, whilst we are treating of the laws which regulate natural prices, natural wages, and natural profits, effects totally independent of these accidental causes.

The competitive process Ricardo describes here harkens back to the old deer-beaver example and forward to modern competitive analysis. The natural prices are defined in terms of equal rates of return – adjusted for any ‘real or fancied’ advantages – and are invariant to any ‘temporary’ or ‘accidental’ causes. Market prices can be affected by all sorts of these causes, but ultimately the desire to obtain the highest return will force them back to the natural price.

What does all of this old stuff have to do with modern financial markets and assets? Surprisingly, the answer is quite a bit. First, the debate over value is of the utmost importance in some investment strategies. In later chapters, we will see that so-called *fundamental investors* still disagree on how to define the value of an asset, in anything less than the most superficial sense. As shown shortly, a fairly superficial treatment can be given to valuing any asset. The problem arises when one attempts to make the treatment applicable.

Second, the distinction between value and price is absolutely crucial in determining investment strategies, understanding markets, and in a host of other ways. During a raging bull market for example, market price tends to dominant the discussion, while value tends to get downplayed. Actually, the distinction will often not exist in the minds of investors any longer during such a period. The ‘value’ of an asset is said to be simply what price it can be sold for – e.g., ‘how much the next sucker will pay for it’. On the other hand, in a volatile market or bear market, one hears much more about how prices are not reflecting ‘true’ or ‘fundamental’ or ‘intrinsic’ value. Recently, we have observed just how important this distinction can be. For example, in August of 2007, it was often reported that investment funds could not value their holdings of complex securities. Moreover, during this period, a money management firm requested a hold on redemptions because it could not value some of its assets. In some cases, the inability to value assets meant that there were simply no buyers. Consider what it would mean if someone attempted to sell their home at a time when there were absolutely no home buyers around. In other cases, like the money management firm, it meant that if they had to value their assets at current market prices, then they would be insolvent. Now, we do not hear about such problems during a booming market time. Few appear to have a problem declaring the value of their assets to be the market price when prices are shooting upward. What people really seem to be saying during the downturns is that the
market price temporarily does not reflect the underlying value of the asset, but the market price will certainly gravitate back towards its value. In other words, the same process that Smith and Ricardo assumed occurred when market prices deviated from natural prices (or, value).

Third, the competitive process noted by Smith and Ricardo remains a pillar of modern portfolio theory and a tool in modern fundamental valuation. Modern portfolio theory is based on a notion of equilibrium defined as a position where the risk adjusted (e.g., ‘real’ or ‘fancied’ advantages) rate of return are equal for all assets – just the definition of natural price all over again. What the modern theory does is flesh out what ‘risk adjusted’ actually means. Modern valuation techniques rely heavily on the notion that competition within and between industries will eventually wipe out any abnormal profits – the process of moving from woolen to silk in Ricardo’s example.

3.2 Forward Looking Valuation

The old ideas have proven their longevity on several fronts. However, there is one crucial place in which they have had to be overthrown. It is crucial in the sense of changing the way we conceptualize the economy and economic activity. Consider the old definition of value as being the amount of labor embodied in the production of a commodity. Now, why labor, among all other inputs, was given the position of priority need not concern us. Defining value in terms of labor embodied is really just a way of thinking about value (and, the natural price) from an input perspective. Value is determined by the amount of inputs, whatever the units of these inputs, required to produce the thing. If we measure the inputs in monetary units, then value was simply the cost of production plus the normal rate of return. Here we have the conceptualization of economic activity. Value is backward looking in the sense that we add up what happened in the past to get to the present. However, financial economics and finance conceptualize economic activity by looking into the future and discounting it back to the present. As a brief example, imagine that you are considering investing in either asset A or asset B. Asset A is a brand new factory that took years to construct at great cost (money and resources), which will produce typewriters. Asset B is an internet dating company run by one person from his basement with a few computers. From an investment standpoint, it probably does not matter much how much it cost to produce either asset. What really matters is the potential profit each asset may generate in the future. Hence, financial economics and finance has a forward looking view of valuation.

If value is thought of as being based on the future, then the present value of an asset can change for all sorts of reasons – none of which has much to do with what happened in the past. A generic, or superficial, definition of value can be stated as the following: the value of an asset is the risk adjusted present value of all future net benefits. Beginning with the last, we will discuss each underlined portion of this definition separately.

The future net benefit of an asset is surprisingly difficult to pin down. On the one hand, there may be intangible (or, non-quantifiable) net benefits of an asset. For
example, a house is an asset with part of the net benefit being the price appreciation and another part being the enjoyment of living in it. The price appreciation would have to be estimated, but so would the enjoyment aspect (e.g., maybe you didn’t realize your neighbors would call the police every time you held a party). On the other hand, limiting the analysis to assets where the net benefit is always quantifiable holds out problems as well. Take the case of investing in a share of a corporation (i.e., stock). As part owner of the corporation, what are your future net benefits? Note we are not attempting to quantify anything yet, just arrive at what should be quantified. One answer might be the profits (or, earnings, net income, etc.) of the corporation. Profits are simply revenues minus expenses, what could be simpler? According to accounting principles, revenues include sales made on credit, which may turn out to be bad credit and never paid. Accounting principles do not count capital expenditures (e.g., new plant and long-lived equipment) as expenses at the time of purchase. Rather, the capital expenditures are expensed as they depreciate over time. At this point, maybe you are thinking that profits should mean cash coming in minus cash going out. This would seem to be very reasonable. However, consider that huge capital expenditure again. Would you really like to say that because the corporation investment $100 million in a new plant with the latest technology in order to drive out its competitors, they were making less profits this year (i.e., the net benefit of the asset was lower)? Would we not be better off attempting to match expenses to the revenues generated from them? This is what the accounting principals attempt to do. Which is correct? We will take up this question much later. For now, it is important to realize that our generic definition of value may have difficulties in application.

The present value concept underlies just about everything we do in valuing assets, thereby making its way through nearly all financial transactions. The basic idea of present value is not very difficult. The simplest case is to think of what happens with a bank account. Suppose you put $100 in a bank account that paid 10% interest for one year. At the end of the one year, you could withdraw $110 – the $100 investment and the $10 interest (or, net benefit). We can translate this into simple math.

\[
100(1+.10) = 100 + 10 = 110
\]

Now, if this bank account existed, what would receiving $110 in a year be worth to you now? In other words, supposed you were offered a contract that would pay you $110 in one year, how much would you pay for the contract? The answer of course is no more than $100. Thus, the present value of $110 a year from now is $100 – given the 10% interest rate. Again, we can translate this into math by a simple rearrangement of what we did before.

\[
100 = \frac{110}{1 + .10}
\]

\[
4 \text{ For those unfamiliar with Generally Accepted Accounting Principles (GAAP), do not fret. We will spend time covering the accounting principles later.}
\]
If you had deposited the money for two years, your $100 would have grown to $121.

$$100(1+.10)(1+.10) = 100(1+.10)^2 = 121$$

Flipping the question, we could ask how much would you pay for a contract that would deliver $121 two years from today? Given the 10\% interest rate, we would pay no more than $100 for the contract. Thus, the present value of $121 two years from now is $100.

In a narrow sense, the concept of present value illustrates the time value of money. As long as we can earn something (e.g., interest) on money today, then money in the future will be worth less at the present time. We can state present value more formally in the following general way.

$$(3.1) \quad PV = \frac{NB_1}{(1+k_1)^1} + \frac{NB_2}{(1+k_2)^2} + \frac{NB_3}{(1+k_3)^3} + \ldots$$

where,

- $PV$ is Present Value
- $NB$ is Net Benefit
- $k$ is the discount rate (more on this shortly)
- subscripts denote the time dimension (e.g., 1 year, 2 year, etc.)

In our previous examples, the net benefit was the payment made to us by the bank and the discount rate was merely the interest rate on the deposit. Equation (3.1) is the general form of our previous examples.

The present value formula appears very simple. However, we have already seen that defining the future net benefit can be tricky when it comes to applications. Even trickier is the mysterious discount rate. Before going too far, we should note that the discount rate can be very subjective. Consider a friend of yours offering to make a pie that you just love. The twist, though, is that the offer is to make you one pie today or one pie a year from today.\(^5\) Whether human nature or socialization, most people seem to require a reward (or, bribe) in order to forgo present enjoyment for future enjoyment. In other words, most people require more of a good thing in the future to give it up today. Suppose then, you counter offer by agreeing to one and a half pies a year from today. What is your subjective discount rate (i.e., how do you discount the future)?

$$1\text{pie} = \frac{1.5\text{pies}}{(1+k)} \rightarrow k = .5 = 50\%$$

The subjective nature of the discount rate can be very important for certain types of analysis. The classical economists, such as Adam Smith and David Ricardo, were

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\(^5\) Notice, your very good friend is not charging you for the pie, thus we have gotten rid of the time value of money basis of the discount rate. Also, you might think of the offer being either today or tomorrow, or either today or in a week, to make it more realistic – we use one year just for ease of calculation.
concerned with how to entice people to give up present enjoyments (e.g., consumption goods) so that resources could be devoted to producing additional plant and equipment (e.g., capital goods). In modern terminology, the classical economists were concerned about how individuals in the society discounted the future.  

The determination of the appropriate discount rate to use in present value calculations is intimately connected with adjustments for risk. It is possible to make risk adjustments to the net benefit component of the present value formula. For instance, if you were not quite sure whether you were going to get $110 next year from your bank deposit, then you could use an expected net benefit calculation – based on probabilities. However, it is more common to make risk adjustments by changing the discount rate. Suppose, for instance, a friend asks for a loan with the agreement of paying you back $110 in one year. Now, as before, you could loan your bank $100 today and receive repayment of $110 a year from now (after all, a bank deposit is simply a loan to the bank). Would you be willing to loan your friend a $100 today in exchange for $110 in a year? The answer is probably a big no. After all, your friend is probably not as credit worthy as the bank. You may want to raise the discount rate you use to lend to your friend above what you require from the bank. Suppose, after analyzing your friend’s finances, you require a 16% discount rate in order to make the loan to your friend, then you would only lend your friend about $95.

\[
95 \cong \frac{110}{(1+.16)}
\]

By raising the discount rate, you are lowering the present value of the $110 from your friend as compared to the bank. This is an important result in any present value calculation. The discount rate and present value always move in opposite directions for any given net benefit.

The tricky business of determining what discount rate should be used in a present value calculation can now be seen. How is one supposed to come up with a discount rate to use in any given situation? It is easy enough to say that one should use a higher discount rate for a loan to a friend than a loan to a bank. However, how much higher should it be? What discount rate should be used when valuing a corporation? Again, we may think that it should be higher for a corporation than for say a loan to the federal government, but how much higher? We arrive back at our conclusion that applying our generic definition of the value of an asset is difficult. The classical economists had recognized that equilibrium would be established when the risk adjusted rates of returns

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6 Part III takes up this issue in greater detail. The focus will be on understanding how financial markets coordinate the intertemporal allocation of resources. We will see that this type of coordination tends to have many more pitfalls than the efficient allocation of resources within a static framework.

7 Of course, all of this could be said in much simpler terms. Really, you are just charging a higher interest rate (16%) to your friend than you do for the bank (10%). The apparent awkward wording in the text anticipates more sophisticated financial instruments. In addition, the wording emphasizes – possibly more than necessary - the distinction between discount rates and interest rates. Discount rates are like interest rates in terms of being pure numbers. However, the two concepts have different interpretations.
for various endeavors would be equal. However, they remained pretty silent on just how to do the actual risk adjustments. Modern portfolio theory has done much to help us flesh out this issue, though disagreements remain.

3.3 The Value of Bonds

Valuing a bond can be accomplished by applying the generic definition of value. A bond is simply a loan to the issuer. An issuer of a bond can range from the federal government to a corporation, and yes even a small private liberal arts college in the Midwest. The fact that a bond represents a loan makes it similar to a bank deposit (i.e., a loan to a bank) and a loan to a friend. The most important difference between a bond and say a loan to a friend is that there normally is an *active secondary market* for bonds. A secondary market is simply where used (or, previously issued) assets are traded. The market for used cars is an example of a secondary market. In fact, most of the trading in bonds (and, as we shall see, for stocks) is done in the secondary market. Thus, the issuer of the bond may have received the loan many years ago, while the title to the loan is being transferred from one hand to another. An *active* secondary market means that the volume of trading is large. An active market ensures that someone wanting to sell the asset will be able to at close to the last price quoted. The rest of the section demonstrates how to *begin* valuing two of the most typical types of bonds.

Bonds come with a variety of characteristics. The characteristics of bonds typically turn on how repayment and interest will be paid. One of the simplest types of bonds is the discount bond. A discount bond makes one payment at the end of the loan agreement. For example, a discount bond may state that it will pay the holder of the bond $1,000 on June 30th, 2008. The $1,000 is the *face value* of the bond – not to be confused with the ‘value of the bond’. The date is referred to as the date of *maturity* – or, when the loan ends. If, for ease of application, we assume that June 30th is one year away, then we can set up the formula for valuing this bond.

\[
V^D = \frac{1,000}{(1 + k)^1}
\]

where,
\[
V^D \text{ is the value of the bond (debt)}
\]
\[
k \text{ is the appropriate discount rate}
\]

If we believed that an 8% discount rate is appropriate for this bond, then the value of the bond would be roughly $926. On the other hand, if 6% was a more appropriate discount rate, then the value of the bond would be roughly $943. Again, notice the inverse relationship between the discount rate and value of the asset. Suppose the maturity date had actually been June 30th, 2013 – five years away. We can determine the value of the bond for each of the assumed discount rates.

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8 We have recently seen that there are times when these markets can cease to function. People who thought they would be able to easily sell their particular bonds had great trouble in finding a buyer. This can happen in any type of secondary market (e.g., housing) for a variety of reasons.
The time value of money has now come into view. Given the face value of $1,000 and discount rate of 8%, the value of the bond went from $926 for a one year bond to $681 for a five year bond. That is, as the $1,000 we receive gets pushed further into the future we value it less today.

How one comes up with the appropriate discount rate to use is only slightly less difficult for a bond than many other assets. Bonds will typically come with a credit rating made by some agency (e.g., Moody’s, etc.). The credit rating is based on an analysis of the financial condition (present and future) of the issuer. A low (high) credit rating will lead people to use a higher (lower) discount rate. We can reverse engineer (a fancy term for rearranging an equation) the valuation in order to discover what discount rate ‘the market’ is using. For example, suppose our $1,000, 5-year discount bond is currently selling for $700, then we can solve for the discount rate.

$$V_D = \frac{1000}{(1+.08)^5} = 681$$

$$V_D = \frac{1000}{(1+.06)^5} = 747$$

Solving for $k$ we get the following.

$$k = \left( \frac{1000}{700} \right)^{\frac{1}{5}} - 1 = .0739 = 7.39\%$$

Here, we see that the market is ‘valuing’ the bond at $700 and discover a 7.39% discount rate being used. Again, one should be careful about the terminology here. It is true that the 7.39% is the interest rate the buyer of the bond would earn if they hold it until maturity. However, this is not necessarily the interest rate the issuer is paying. If this purchase is on the secondary market, then the bond may have been issued many years ago and more important at a price other than $700. Furthermore, for the buyer, the 7.39% may not be the rate of return earned on the bond. Suppose, the buyer turns around and sells the bond a month later for a price of $770. The buyer’s rate of return would be 10%.

$$Rate\ of\ Return = \frac{770 - 700}{700} = .10 = 10\%$$

As with so much in this chapter, we will come back to this issuer later and flesh out the details. Our purpose here is to begin the process.
Care will need to be taken concerning terminology for a bit. Once we get used to the terminology, then no real harm will be done by using interest rate more often.

Unlike a discount bond, a coupon bond makes periodic interest payments during the life of the loan. The face value will still refer to the last payment. There will be a maturity date as well. However, there will also be a coupon rate stating the percentage of the face value that will be paid out as interest periodically (annually, semi-annually, etc.). We will often assume that interest (i.e., the coupon payment) is paid annually in order to simplify the calculations. For example, suppose we want to value the following bond.

Face Value = $1,000
Time to Maturity = 3 years
Coupon rate = 5%

The 5% coupon rate implies that the issuer will make annual payments to the holder of the bond in the amount of $50 per year. If, given the characteristics of the issuer, we believe a 7% discount rate is appropriate the value of the bond would be the following.

\[ v^D = \frac{50}{(1+.07)^1} + \frac{50}{(1+.07)^2} + \frac{50}{(1+.07)^3} + \frac{1,000}{(1+.07)^3} = \]

\[ = 46.73 + 43.67 + 40.81 + 816.30 = 947.51 \]

We see the time value of money once again as the $50 coupon (interest) payment is received further in the future. What would happen if the appropriate discount rate dropped to 4%? The value of the bond would be $1,028.75. The appropriate discount rate may have dropped because the issuer was more credit worthy than before (e.g., received a big contract, sold off unprofitable parts of the business, etc.). The drop in the discount rate has meant that we value the bond more now.

It is possible, but not easy without a financial calculator or computer, to reverse engineer a coupon bond. For example, if the bond was currently selling for $980, then we might solve for the discount rate being applied to the bond.

\[ 970 = \frac{50}{(1+k)^1} + \frac{50}{(1+k)^2} + \frac{50}{(1+k)^3} + \frac{1,000}{(1+k)^3} \]

Just by glancing at the above equation we see that it will not be an easy task to solve for the discount rate. In fact, we cannot solve for the discount rate. We would have to use a trial-and-error process in order to pin down the discount rate. For example, at a price of $970, we know that the discount rate will be lower than 7% but higher than 4% (how do we know this?). Most spreadsheets have a function that will solve for the discount rate. The discount rate for this bond selling at a price of $970 is 6% (you should verify this by plugging in 6% in the above).
The discount rate that sets the present value of the future payments (coupon and face value) equal to the price of a bond is called the yield to maturity. Typically, what people refer to as the interest rate on a bond is the yield to maturity. Due to the difficulty of calculating the yield to maturity (at least prior to financial calculators and computers), people have sometimes used an approximation known as the current yield. The current yield is simply the coupon payment divided by the current price of the bond. In the previous case, the current yield would be the following.

\[
\text{Current yield} = \frac{50}{970} = 0.0515 = 5.15\%
\]

The current yield is still reported in most financial news publications. However, it should be clear that it is only an approximation to the interest rate of most significance (i.e., yield to maturity).

3.4 The Value of Stocks

Stocks tend to be more difficult than bonds to value. The first difficulty arises from identifying just what the net benefit should be – not just the numerical value. A stock represents part ownership in a corporation. The owners have a claim on the profits (or, net income, earnings) the corporation generates. Understood from this perspective, the net benefit would be the share of the future profits the stock represents. On the other hand, the owners have a claim to the equity (or, net wealth) of the corporation at any given time. The net benefit from owning a stock could be the claim to the future equity of the corporation.

There are a variety of perspectives that lead to differing notions of the net benefit of a stock. The current section develops a simple, but very useful, approach to defining net benefit. One may think of the net benefit of a stock as ultimately residing in the cash flow going to stockholders. This cash flow comes in the form of a dividend payment. A dividend is the portion of the profits earned that the corporation pays to its owners (i.e., stockholders). Many corporations choose not to pay dividends, especially in their early or start-up phases. If the corporation, for example, has a better – in terms of higher rate of return - place to invest the potential dividends than the stockholder, then it would seem to be appropriate not to pay a dividend. Alternatively, if the corporation suffers losses, then a dividend may not be paid.

Valuation of stocks based on defining net benefit as dividend can be accomplished in different ways. The approach followed here will be the Gordon Model (or, Dividend Discount Model). We can present the basic formulation very simply.

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10 The yield to maturity, as we will see, is a little more complicated than discussed so far. It presumes that the periodic payments (i.e., coupon payments) are able to be reinvested at the yield to maturity. This condition will very likely not be met.

11 The definition of cash flow to the stock holder could include stock repurchases as well as dividends.
\[ V^E_0 = \frac{D_1}{(1+k)} + \frac{D_2}{(1+k)^2} + \frac{D_3}{(1+k)^3} + \ldots + \frac{P_n}{(1+k)^n} \]

where,
- \( V^E \) is the value of the stock (equity) at the present time
- \( D_i \) is the dividend paid in period \( i \)
- \( k \) is the appropriate discount rate
- \( P_n \) is the price of the stock when eventually sold

First, notice that we have just applied the present value formula. Second, notice that we have a circularity problem – a problem that often arises in valuation. We are attempting to arrive at the value of an asset independent of price. However, in the above, we are saying that the current value of the stock partially depends upon the expected price in period \( n \) (when we sell it). At this point we will gloss over this problem and assume that by the time we wish to sell the stock its price will be the same as its value.\(^\text{12}\) Upon making this assumption, we can rewrite the above.

\[ V^E_0 = \frac{D_1}{(1+k)} + \frac{D_2}{(1+k)^2} + \frac{D_3}{(1+k)^3} + \ldots + \frac{V^E_n}{(1+k)^n} \]

The Gordon model takes the above and adds the assumption of constant dividend growth. The assumption may seem unrealistic. It is, though not drastically so. Many corporations keep their dividend the same (zero growth) for long stretches of time or target a particular growth rate. We can now begin to rework the formulation. First, notice that the value of the stock at time \( n \) will be the present value of the future dividends. The formula becomes an infinite series.

\[ V^E_0 = \frac{D_1}{(1+k)} + \frac{D_2}{(1+k)^2} + \frac{D_3}{(1+k)^3} + \frac{D_4}{(1+k)^4} + \frac{D_5}{(1+k)^5} + \ldots \]

Second, assume the dividend grows at a constant rate \((g)\) forever.

\[ V^E_0 = \frac{D_0(1+g)}{(1+k)} + \frac{D_0(1+g)^2}{(1+k)^2} + \frac{D_0(1+g)^3}{(1+k)^3} + \frac{D_0(1+g)^4}{(1+k)^4} + \frac{D_0(1+g)^5}{(1+k)^5} + \ldots \]

Finally, the above equation can be simplified to

\[ V^E_0 = \frac{D_1}{k-g} \]

\(^\text{12}\) For many, the fudge in the text will be hard to swallow. It should. We will return to this problem later. Remember, for now, we are just attempting to introduce the ideas.
and we arrive at the Gordon Model. According to this model, in order to value a stock, we need to estimate next period’s dividend, the growth rate of the dividend, and arrive at the appropriate discount rate. After arriving at the value, the active investor can follow some simple rules: buy when value exceeds market price, do not buy (or, short) when market price exceeds value.

It is the simplicity of the Gordon Model that makes it useful. Although many value investors have moved beyond the model, it can still serve as a useful starting point. For example, if you read the stock tables in a newspaper, then you will mostly likely observe a column for the P/E ratio (i.e., price-earnings ratio). Some fundamental investors will look to purchase stocks with low P/E ratios. The Gordon Model can help us understand why. We can begin by dropping the notation of value and use the more common price terminology. Next, we recognize that dividends are simply the portion of earnings ($E$) that get paid out to stockholders – define the portion as $b$.

$$P_0 = \frac{bE_1}{k - g} \Rightarrow$$

$$\frac{P_0}{E_1} = \frac{b}{k - g}$$

The P/E ratio depends up the payout rate (i.e., what percentage of earnings are paid out as dividends), the growth rate of dividends (and, assuming a constant payout ratio, the growth rate of earnings!), and the appropriate discount rate. Upon observing a P/E ratio for a particular stock, we can begin to discover what assumptions would need to be made in order to justify it. Further substitutions can be made to the model (e.g., replacing growth with what it depends upon) and increase its usefulness, but for now it helps put us on the correct track for understanding the valuation of a stock.

One final rearrangement of the original Gordon Model is useful at this point. The model can be solved for the discount rate. When this is done, the discount rate is reinterpreted as an expected rate of return ($r$) on the stock.

$$r = \frac{D}{P} + g$$

The rate of return depends upon the dividend yield ($D/P$) – often reported in stock tables – and the expected growth rate of dividend (and, earnings). For example, a stock currently priced at $50 and paying a $5 dividend with expected growth of 4% per year has an expected rate of return of 14% (plug in the numbers and see). If the stock price should fall to $45 – assuming the dividend and growth remain the same – the expected rate of return will rise to just over 15% (the dividend yield rises to 11.1%). Though all of

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13 There are actually two P/E ratios. One is a lagging P/E ratio and uses the past earnings. The second is the forward P/E ratio and uses the estimate for next period earnings.
this is useful, we have made several assumptions and glossed over some difficulties – e.g., we have not even mentioned how to arrive at an appropriate discount rate to be used in the Gordon Model. We will return to the question of the valuation of a stock in much more detail later.

3.5 Bachelier’s Drunken Walk

Have you ever tried to help a friend walk home after they have had too much to drink? If so, you probably spent much of your time guessing which way and when they were going to stumble next. You find yourself quickly reaching for your friend as they sway one way, then the next. You have to move quickly after they begin to sway because you cannot predict where they will go next. In 1900, a wrong mathematician named Louis Bachelier demonstrated that this same process worked in the stock market. The conclusion had far reaching consequences for how some think about modern asset markets. Bachelier’s work, though unknown at the time, provided the foundation for what has come to be called the efficient market hypothesis.\footnote{The efficient market hypothesis comes in at least three forms. We will discuss all three later. For now, as with this entire chapter, our goal is introduce the issues to be developed later.}

Suppose that asset prices were not like your friend’s drunken walk home. Suppose that asset prices were perfectly predictable. For example, suppose you found that stocks with low P/E ratios today (say, below 10) would have higher prices tomorrow. What would you do today? I am just guessing here, but you might decide to buy those stocks today. If so, then what would happen to their prices today? Of course, the price would go up, causing the P/E ratio to rise. When tomorrow does come, the stock price would have already moved to the predicted price. In fact, if it had not risen, there would be evidence that something strange was going on (e.g., an inefficiency existed in the market). In the opposite case, if stocks with high P/E ratios today had lower prices tomorrow, then you would not buy those stocks. In fact, if the rule held true, then you would want to \textit{short} the stocks.\footnote{We will discuss shorting stocks later (or, short-selling). For now, shorting simply means selling stocks you don’t own. The mechanics are fairly simply. You borrow the stocks from a broker and sell them. If you are right, then the price of the stocks will fall in the future. When the stock price does fall, you buy it back and repay your broker – making a nice little profit for yourself. Rather than using the mantra “buy low and sell high” you use “sell high and buy low.”}

The stock price would come down today, implying that by tomorrow the opportunity had passed.

In the financial literature, the process above is termed a random walk. In an efficient market, asset prices should follow a random walk. Hence, attempts to predict stock prices are for the most part pointless. Formally, the inability to predict stock prices, can be stated in the following manner.

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E(p_1 | I_0) = p_0
\]

where,

\( p_1 \) is the price at time 1 (e.g., tomorrow)
\( p_0 \) is the price at time 0 (e.g., today)
I_0 is all the information available at time 0 (e.g., today)

In words, the expected (the operator E) price of the stock tomorrow given (the operator |) the information available today is equal to the price of the stock today. The ‘information’ may include only past stock prices or just about anything that would be important (e.g., past earnings, announcement of a new contract, GDP, etc.)\(^{16}\) The equation summarizes the logic described earlier. In short, whatever information is available today (e.g., P/E ratio) will be included in the price today, thus leaving the current price as the best estimate of the future price. Since any new information that comes to the attention of investors must be random, then the movement of stock prices will be random.

We have already stated that if stock prices follow a random walk, then attempts to predict them will be pointless (at least, for the most part). The statement has greater significance than might first appear. It means that value and price are nearly always the same. Hence, there is really no point in undertaking a valuation exercise (such as in the previous section). If one accepts that market price and value are one and the same, then active investment strategies will not be profitable – except by pure luck. The *Wall Street Journal* used to test this idea by comparing the results of picking stocks by throwing darts at a list of corporations (i.e., choosing randomly) with stocks picked by so-called ‘experts’ in the field.

Even if one accepts the random walk movement of asset prices in general, there is still much to learn in financial economics and finance. Modern portfolio theory, for example, chooses to focus on how best to construct an entire portfolio of assets, rather than how to pick individual assets. Portfolio theory combined with the random walk does suggest strategies for the active investor as well. The negative comments concerning active strategy above have been qualified (i.e., ‘for the most part’) to allow for particular situations. For example, not all stocks are treated equally by professionals. Some stocks are covered (or, studied) by many professional analysts, suggesting the relevant information is probably contained in the market prices. However, there are stocks that fly under the radar in the sense that analysts do not cover them. The market prices for stocks less heavily studied may not incorporate all relevant information, providing an opening for the active investor (or, trader). Furthermore, if we open up the definition of assets to include those not traded on organized markets (e.g., NYSE, NASDAQ, etc.), then valuation methods may come back into play. A few of the large university endowment funds have been able to earn higher than normal returns by investing in non-traditional assets (e.g., private equity, land, etc.). This type of investment is really in search of inefficient markets.

### 3.6 Conclusion

{need to write}

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\(^{16}\) The information set typically determines what form of the efficient market hypothesis is being tested.