Notes on the Cost of Capital

1. Introduction

We have seen that evaluating an investment project by using either the Net Present Value (NPV) method or the Internal Rate of Return (IRR) method requires a determination of the firm’s cost of capital. Again, the terms cost of capital, required rate of return, hurdle rate, and weighted average cost of capital (WACC) tend to be used interchangeably. This terminological point has been emphasized because it tends to cause confusion when first learning corporate finance. The important point is that these terms refer to the ‘appropriate’ discount rate to be used when evaluating investment projects and when attempting to value a firm. In either case, it is necessary to determine the present value of a future flow of net benefits (e.g., profits, cash flows, etc.). The present value calculation requires a discount rate to be used and the question naturally arises as to what the appropriate discount rate should be. The cost of capital is this discount rate.

The current set of notes demonstrates how a corporation arrives at its cost of capital. Thinking of cost of capital in terms of the required rate of return, we observe that what is really necessary is to discover what rate of return the current credit-holders and shareholders require. A useful way to visualize the thinking is given below.

The diagram illustrates that the firm may raise financing in the financial markets by issuing debt (liabilities) or new shares (equity). The financing raised will add to the firm’s net financial assets (in this case, cash). The firm will use the cash raised to invest in additional net operating assets (e.g., property, plant, & equipment). Depending upon the firm, the net operating assets could be new equipment, new manufacturing plants, patents, etc. In short, these are the assets the firm uses to do what it is in business to do. The purchase of the operating assets constitutes a capital expenditure by the firm and outflow of cash. If all goes well, the firm will be able to generate additional earnings from their operations and capital expenditure, which returns to the firm by increasing their net financial assets – thus, a cash inflow. The firm will use the financial assets (cash) to make payments to the financial markets in the form of interest and principal on debt to credit-holders and/or dividends to shareholders.

The goal of the current notes is to understand how the financial markets signal the cost of capital to the firm, thereby informing the capital expenditure decision. It is important to keep in mind throughout that this signal will be present regardless of how the additional investment (i.e., capital expenditure) will be financed. Thus, in our simple story, the signal (cost of capital) will be present even if the firm had chosen to make the capital expenditure in additional net operating assets out of their existing net financial assets (thus, retained earnings). The cost of capital is determined by the current credit-holders and shareholders, not the ones that had made the purchase of the original issue of new shares or debt. This should become clearer as we get into the details of arriving at the cost of capital. We must now begin that process by seeing how to determine the required rate of return from credit-holders and shareholders. We will begin with credit-holders first and assume the firm has issued bonds as debt (the case of bank loans would be even simpler).

2. The Cost of Debt --- Bonds

Bonds come with a variety of characteristics. The characteristics of bonds typically turn on how repayment and interest will be paid. One of the simplest types of bonds is the discount bond (sometimes referred to as zeros). A discount bond makes one payment at the end of the loan agreement. For example, a discount bond may state that it will pay the holder of the bond $1,000 on November 1st, 2011. The $1,000 is the face value of the bond – not to be confused with the ‘value of the bond’. The date is referred to as the date of maturity – or, when the loan ends. If, for ease of application, we assume that November 1st is one year away, then we can set up the formula for valuing this bond.

\[ V^D = \frac{1,000}{(1 + k)^1} \]

where,

- \( V^D \) is the value of the bond (debt)
- \( k \) is the appropriate discount rate

If we believed that an 8% discount rate is appropriate for this bond, then the value of the bond would be roughly $926. On the other hand, if 6% was a more appropriate discount
rate, then the value of the bond would be roughly $943. Again, notice the inverse relationship between the discount rate and value of the bond. Suppose the maturity date had actually been November 1st, 2015 – thus, five years away. We can determine the value of the bond for each of the assumed discount rates.

\[ V^D = \frac{\$1,000}{(1 + .08)^5} = \$681 \]

\[ V^D = \frac{\$1,000}{(1 + .06)^5} = \$747 \]

The time value of money has now come into view. Given the face value of $1,000 and discount rate of 8%, the value of the bond went from $926 for a one year bond to $681 for a five year bond. That is, as the $1,000 we receive gets pushed further into the future we value it less today.

How one comes up with the appropriate discount rate to use is only slightly less difficult for a bond than many other assets. Bonds will typically come with a credit rating made my some agency (e.g., Moody’s, Standard & Poors, etc.). The credit rating is based on an analysis of the financial condition (present and future) of the issuer. A low (high) credit rating will lead people to use a higher (lower) discount rate. We can reverse engineer (a fancy term for rearranging an equation) the valuation in order to discover what discount rate ‘the market’ is using. For example, suppose our $1,000, 5-year discount bond is currently selling for $700, then we can solve for the discount rate.

\[ 700 = \frac{1,000}{(1 + k)^5} \]

Solving for \( k \) we get the following.

\[ k = \left( \frac{1,000}{700} \right)^{1/5} - 1 = .0739 = 7.39\% \]

Here, we see that the market is ‘valuing’ the bond at $700 and discover a 7.39% discount rate being used. Again, one should be careful about the terminology here. It is true that the 7.39% is the interest rate the buyer of the bond would earn if they hold it until maturity. However, this is not necessarily the interest rate the issuer (i.e., the firm) is paying. If this purchase is on the secondary market, then the bond may have been issued many years ago and more important at a price other than $700. Furthermore, for the buyer, the 7.39% may not be the rate of return earned on the bond. Suppose, the buyer turns around and sells the bond a month later for a price of $770. The buyer’s rate of return would be 10%.
Rate of Return = \( \frac{770 - 700}{700} = .10 \rightarrow 10\% \)

Care will need to be taken concerning terminology for a bit. Once we get used to the terminology, then no real harm will be done by using the term interest rate instead of discount rate.

Unlike a discount bond, a coupon bond makes periodic interest payments during the life of the loan. The face value will still refer to the last payment. There will be a maturity date as well. However, there will also be a coupon rate stating the percentage of the face value that will be paid out as interest periodically (annually, semi-annually, etc.). We will often assume that interest (i.e., the coupon payment) is paid annually in order to simplify the calculations. For example, suppose we want to value the following bond.

- Face Value = $1,000
- Time to Maturity = 3 years
- Coupon rate = 5%

The 5% coupon rate implies that the issuer will make annual payments to the holder of the bond in the amount of $50 per year (i.e., 5% of the $1,000 face value). If, given the characteristics of the issuer, we believe a 7% discount rate is appropriate the value of the bond would be the following.

\[
V = \frac{50}{(1+.07)^1} + \frac{50}{(1+.07)^2} + \frac{50}{(1+.07)^3} + \frac{1000}{(1+.07)^3} = 947.51
\]

We see the time value of money once again as the $50 coupon (interest) payment is received further in the future. What would happen if the appropriate discount rate dropped to 4%? The value of the bond would be $1,028.75. The appropriate discount rate may have dropped because the issuer was more credit worthy than before (e.g., received a big contract, sold off unprofitable parts of the business, etc.). The drop in the discount rate has meant that we value the bond more now.

It is possible, but not easy without a financial calculator or Excel, to reverse engineer a coupon bond. For example, if the bond was currently selling for $980, then we might solve for the discount rate being applied to the bond.

\[
970 = \frac{50}{(1+k)^1} + \frac{50}{(1+k)^2} + \frac{50}{(1+k)^3} + \frac{1000}{(1+k)^3}
\]

Just by glancing at the above equation we see that it will not be an easy task to solve for the discount rate. In fact, we cannot solve for the discount rate directly. We would have to use a trial-and-error process in order to pin down the discount rate. For example, at a
price of $970, we know that the discount rate will be lower than 7% but higher than 4% (how do we know this?). Most spreadsheets have a function that will solve for the discount rate. The discount rate for this bond selling at a price of $970 is 6% (you should verify this by plugging in 6% in the above). Notice how similar this is to the Internal Rate of Return (IRR) from the previous set of notes. In that case, the internal rate of return was merely the discount rate that made the present value of the future cash flows from an investment project just equal to the current cost of the project. In this case, we are attempting to solve for the discount rate that makes the present value of the future payments (coupon and face value) just equal to the current selling price of the bond.

The discount rate that sets the present value of the future payments (coupon and face value) equal to the price of a bond is called the **yield to maturity**. Typically, what people refer to as the interest rate on a bond is the yield to maturity. Due to the difficulty of calculating the yield to maturity (at least prior to financial calculators and computers), people have sometimes used an approximation known as the current yield. The current yield is simply the coupon payment divided by the current price of the bond. In the previous case, the current yield would be the following.

\[
\text{Current yield} = \frac{\$50}{\$970} = .0515 = 5.15\%
\]

The current yield is still reported in most financial news publications. However, it should be clear that it is only an approximation to the interest rate of most significance (i.e., yield to maturity). The **yield to maturity** is often abbreviated as **YTM**.

We have been discussing bonds as if our goal was to value them ourselves. What has this to do with a corporation discovering what rate of return is being required by current credit-holders (or, in our case, bond-holders)? Everything! In fact, we have already answered the question. The rate of return required by current bond-holders is the yield to maturity (YTM). To see this, assume a corporation issues (i.e., sells, or floats) a new bond on January 1, 2010. The bond has the following characteristics (note, we assume a single bond just to make the discussion easier, things would be no different if it were a thousand bonds).

- **Face Value** = $1,000
- **Time to Maturity** = 4 years
- **Coupon rate** = 4%

Furthermore, assume the bond was actually sold for $1,000. In this case, the yield to maturity on the bond would be 4%, same as the coupon rate.

\[
\begin{align*}
1,000 &= \frac{\$40}{(1+.04)^1} + \frac{\$40}{(1+.04)^2} + \frac{\$40}{(1+.04)^3} + \frac{\$40}{(1+.04)^4} + \frac{\$1,000}{(1+.04)^4} \\
&= \$38.46 + \$36.98 + \$35.56 + \$34.19 + \$854.80 = \$1,000
\end{align*}
\]
Now, suppose a year has gone by and the original purchaser of the bond decides to sell it. If everything had remained pretty much the same, then the purchaser should be able to sell it with the same yield to maturity and for a price of $1,000. Double check this in order to see that the three years remaining with $40 coupon payments and a final payment of $1,000 face value will lead to a value of $1,000 as long as the discount rate to be used remained at 4%. But, more interestingly, suppose that during the year the corporation had not done very well. The corporation may have lost an important customer or faced stiffer than anticipated competition, or all sorts of bad things. The point though is that the original purchaser finds that he/she can only sell the bond for a price of $950. What rate of return does the new holder of the bond require?

\[
$950 = \frac{\$40}{(1 + YTM)^1} + \frac{\$40}{(1 + YTM)^2} + \frac{\$40}{(1 + YTM)^3} + \frac{\$1,000}{(1 + YTM)^3}
\]

By solving for the yield to maturity (YTM), we discover what the current required rate of return is for the current bond-holder. Using Excel, we find the yield to maturity to be 5.87%.

\[
$950 = \frac{\$40}{(1 + 0.0587)^1} + \frac{\$40}{(1 + 0.0587)^2} + \frac{\$40}{(1 + 0.0587)^3} + \frac{\$1,000}{(1 + 0.0587)^3}
\]

Thus, when things did not go as well for the corporation and the risk of the debt increased, the price (or, value) of the bond fell causing the yield to maturity to increase from 4% to 5.87%. If things had gone the other way, and the business prospects looked better than a year ago, we may have observed an increase in the price of the bond and decrease in its yield to maturity.

What does this have to do with the corporation? On first glance the answer would be not much. After all, the corporation has merely promised to make the $40 coupon payments and $1,000 face value payment over the course of the 4 year life of the bond to whomever is holding the bond. The fact that the original purchaser of the bond could only sell it for $950 does not change the payments that the corporation must make – only to whom it should write the checks, but not the size of the checks. On the other hand, this change does impact the cost of capital – specifically, in this case, the cost of debt. Bondholders are now signaling to the corporation that they demand a higher rate of return to from the corporation due to the changed circumstances. This will impact the cost of capital the corporation uses in the Net Present Value method and Internal Rate of Return method. It will also impact the present value of the firm itself. Though, we will need to wait to see these implications. For now, we may summarize with the following.

**Cost of Debt for a corporation is the Yield to Maturity**

Yield to Maturity is the discount rate that makes the present value of future payments on a bond just equal to the current selling price of the bond
Fortunately, the yield to maturity for a bond is easily obtained on a daily basis from the financial section of most newspapers. If not, the corporation merely needs to know the characteristics of the bond (e.g., face value, coupon rate, time to maturity) and the current selling price in order to calculate the yield to maturity and know what its bond-holders require for a rate of return. Thus, the current cost of debt is easily obtainable. What about the cost of equity?

3. Cost of Equity --- Stocks

Stocks tend to be more difficult than bonds to value. The first difficulty arises from identifying just what the net benefit should be – not just the numerical value. A stock represents part ownership in a corporation. The owners have a claim on the profits (or, net income, earnings) the corporation generates. Understood from this perspective, the net benefit would be the share of the future profits the stock represents. On the other hand, the owners have a claim to the equity (or, net wealth) of the corporation at any given time. The net benefit from owning a stock could be the claim to the future equity of the corporation.

There are a variety of perspectives that lead to differing notions of the net benefit of a stock. The current section develops a simple, but very useful, approach to defining net benefit. One may think of the net benefit of a stock as ultimately residing in the cash flow going to stockholders. This cash flow comes in the form of a dividend payment. A dividend is the portion of the profits earned that the corporation pays to its owners (i.e., stockholders). Many corporations choose not to pay dividends, especially in their early or start-up phases. If the corporation, for example, has a better – in terms of higher rate of return - place to invest the potential dividends than the stockholder, then it would seem to be appropriate not to pay a dividend. Alternatively, if the corporation suffers losses, then a dividend may not be paid. We will look at two alternative ways of arriving at the cost of equity – or, the current required rate of return by shareholders – in this section. The first is based on determining the present value of the future dividends a corporation will pay. This method emphasizes the present value aspect of stocks and makes clear certain important properties. The second method, more often used, is the Capital Asset Pricing Model or CAPM for short (read as Cap-M).

3.1 Dividend Discount Model

Valuation of stocks based on defining net benefit as dividend can be accomplished in different ways. The approach followed here will be the Dividend Discount Model. We can present the basic formulation very simply.

\[
V_0^E = \frac{D_1}{(1 + k)} + \frac{D_2}{(1 + k)^2} + \frac{D_3}{(1 + k)^3} + \ldots + \frac{P_n}{(1 + k)^n}
\]

where,

1 The definition of cash flow to the stock holder could include stock repurchases as well as dividends.
$V^E$ is the value of the stock (equity) at the present time

$D_i$ is the dividend paid in period $i$

$k$ is the appropriate discount rate

$P_n$ is the price of the stock when eventually sold

First, notice that we have just applied the present value formula. Second, notice that we have a circularity problem – a problem that often arises in valuation. We are attempting to arrive at the value of an asset independent of price. However, in the above, we are saying that the current value of the stock partially depends upon the expected price in period $n$ (when we sell it). At this point we will gloss over this problem and assume that by the time we wish to sell the stock its price will be the same as its value. Upon making this assumption, we can rewrite the above.

$$V_0^E = \frac{D_1}{1 + k} + \frac{D_2}{(1 + k)^2} + \frac{D_3}{(1 + k)^3} + \ldots + \frac{V_n^E}{(1 + k)^n}$$

The Dividend Discount Model takes the above and adds the assumption of constant dividend growth. The assumption may seem unrealistic. It is, though not drastically so. Many corporations keep their dividend the same (zero growth) for long stretches of time or target a particular growth rate. We can now begin to rework the formulation. First, notice that the value of the stock at time $n$ will be the present value of the future dividends. The formula becomes an infinite series.

$$V_0^E = \frac{D_1}{1 + k} + \frac{D_2}{(1 + k)^2} + \frac{D_3}{(1 + k)^3} + \frac{D_4}{(1 + k)^4} + \frac{D_5}{(1 + k)^5} + \ldots$$

Second, assume the dividend grows at a constant rate ($g$) forever.

$$V_0^E = \frac{D_0 (1 + g)}{(1 + k)} + \frac{D_0 (1 + g)^2}{(1 + k)^2} + \frac{D_0 (1 + g)^3}{(1 + k)^3} + \frac{D_0 (1 + g)^4}{(1 + k)^4} + \frac{D_0 (1 + g)^5}{(1 + k)^5} + \ldots$$

Finally, the above equation can be simplified to

$$V_0^E = \frac{D_1}{k - g}$$

and we arrive at a version of the Dividend Discount Model often referred to as the Gordon Model. According to this model, in order to value a stock, we need to estimate next period’s dividend, the growth rate of the dividend, and arrive at the appropriate discount rate.

It is the simplicity of the Gordon Model that makes it useful. Although many investors have moved beyond the model, it can still serve as a useful starting point. For example, if you read the stock tables in a newspaper, then you will mostly likely observe
a column for the P/E ratio (i.e., price-earnings ratio --- note, in finance, net income is often referred to as earnings). Some investors will look to purchase stocks with low P/E ratios. The Gordon Model can help us understand why. We can begin by dropping the notation of value and use the more common price terminology. Next, we recognize that dividends are simply the portion of earnings \( E \) that get paid out to stockholders – define the portion as \( b \).

\[
P_0 = \frac{bE_1}{k - g} \Rightarrow \frac{P_0}{E_1} = \frac{b}{k - g}
\]

The P/E ratio depends upon the payout rate (i.e., what portion of earnings are paid out as dividends), the growth rate of dividends (and, assuming a constant payout ratio, the growth rate of earnings!), and the appropriate discount rate.\(^2\) Upon observing a P/E ratio for a particular stock, we can begin to discover what assumptions would need to be made in order to justify it.

One final rearrangement of the original Gordon Model is most useful for us. The model can be solved for the discount rate. When this is done, the discount rate \( (k) \) is reinterpreted as an expected rate of return \( (r) \) on the stock. Or, for us, the rate of return required by current shareholders – hence, the cost of equity capital.

\[
r = \frac{D}{P} + g
\]

The rate of return depends upon the dividend yield \( (D/P) \) – often reported in stock tables – and the expected growth rate of the dividend (and, earnings assuming a constant payout ratio). For example, a stock currently priced at $50 and paying a $5 dividend with expected growth of 4% per year implies that shareholders require a rate of return of 14% (plug in the numbers and see). If the stock price should fall to $45 – assuming the dividend and growth remaining the same – the required rate of return (i.e., cost of equity capital) will rise to just over 15% (the dividend yield rises to 11.1%). A corporation can, therefore, use this model to estimate its cost of equity capital by observing the current market price, the current dividend it will pay, and the expected growth of the dividend. Of course, the problem is that the expected growth of the dividend is really inside the head of its shareholders. Though, with guidance from the corporation itself, the model might be helpful. Probably the most important thing to note for our purposes is the relationship between the current stock price and the cost of equity capital. When the stock price rises, current shareholders are signaling to the corporation that they require a lower rate of return (hence, the cost of equity capital falls).

\(^2\) There are actually two P/E ratios. One is a lagging P/E ratio and uses the past earnings. The second is the forward P/E ratio and uses the estimate for next period earnings.
3.2 The Capital Asset Pricing Model (CAPM)

The Capital Asset Pricing Model (or, CAPM for short) leads to one of the most famous and heavily used equations in finance. The derivation of this model requires a great deal of work and encompasses many lines of thought. However, the equation embodying the CAPM is fairly easy and straightforward. In order to see the intuition behind the CAPM, consider how you would come up with an appropriate discount rate to be used in a present value calculation for determining the value of any asset. The appropriate discount rate would most likely be composed of two parts. The first component would be something like the interest rate on a U.S. government bond (i.e., Treasury Bond). A Treasury bond would be risk-free in the sense that the U.S. government is extremely unlikely to default on its debt. The second component would be an addition to the risk-free rate of return to take into account the risk of the specific asset that you were attempting to value.

\[
\text{Appropriate Discount Rate} = \text{Risk-Free Rate} + \text{Risk Premium}
\]

Alternatively, you can think of the above as explaining the rate of return on a particular asset.

\[
\text{Rate of Return} = \text{Risk-Free Rate} + \text{Risk Premium}
\]

In this case, we explain why a particular asset provides a rate of return above a risk-free asset because of the risk of the particular asset. For example, suppose the interest rate (i.e., yield to maturity) on a Treasury bond is 3% and you purchased a stock that gave you a rate of return of 7%. Why did your stock’s rate of return exceed the interest rate on the Treasury bond? The answer is that your stock entailed greater risk than an investment in U.S. Treasuries. To be precise, it carried a 4% risk premium. Suppose your stock, for whatever reason, became less risky then investors would increase the demand for the stock and drive up its price. But, when the stock price goes up, the required rate of return falls (take a look back at the Gordon Model). From the above, we would say that the risk premium fell resulting in a lower rate of return (or, lower appropriate discount rate).

The CAPM utilizes the basic idea that return is composed of a risk-free rate and risk premium. The model turns to the issue of just how to precisely arrive at the risk premium of an asset. In order to discuss it further, we should go ahead and write down the famous equation

\[
E(r) = r_f + \beta (r_M - r_f)
\]

where
- \(E(r)\) is the Expected (the \(E\)) rate of return on an asset
- \(r_f\) is the risk-free rate of return (e.g., interest rate on a Treasury bond)
- \(r_M\) is the rate of return on the Market for all risky assets

\[\text{For those students interested in the theory behind the CAPM and its use, you should take the Money & Capital markets course.}\]
\(\beta\) (pronounced Beta) is a measure of the relationship between variations in the rate of return on an asset and the rate of return on the Market for all risky assets.\(^4\)

\(\beta[r_M - r_f]\) is the measure of the risk premium of the asset.

How does one apply the CAPM to arrive at a measure of the cost of equity capital (hence, the current required rate of return by shareholders)? First, it is necessary to define the risk-free rate of return. This is normally done by using the interest rate on a government bond (such as a Treasury) with a maturity that comes closest to the length of time of a particular investment project. If the investment project will generate earnings for a corporation over the next 10 years, for example, then the interest rate on a 10-year Treasury bond would be used. Second, it is necessary to arrive at the expected rate of return for the stock market as whole for the next ten years. In practice, historical data is typically used for something like the S&P 500 (a stock index comprised of 500 stock listed on the market). Although we would like to know what the rate of return on the S&P 500 would be for the next 10 years, it is probably safer to use the historical rate of return rather than guess what will happen in the future. Third, and this is the really interesting part, the beta (\(\beta\)) must be determined. In fact, the beta will be the only thing that changes between various stocks – the other two variables in the equation remain the same when comparing two different stocks.

The beta of a stock measures its specific risk relative to the market for all risky assets (e.g., the stock market as a whole). Before getting at the intuition, we should keep in mind that risk in finance does not really mean the commonsense definition. In finance, risk can refer to both the upside (or, possible gain) of an investment as well as its downside (or, possible loss). Normally, we think of risk only in terms of the possible downside impact. In finance terms, it is better to think of the term risk as opportunity, which denotes both upside and downside potential. Technically, risk refers to the variation (or, more precisely, the standard deviation) in the rate of return. The intuition behind the beta term can be seen by comparing a stock the moves up or down twice as much as the market with a stock that moves half as much as the market. Thus, the first stock would have a beta of 2, whereas the second stock would have a beta of one-half. For example, if the stock market as a whole went up or down by 6%, then the rate of return on the first stock would go up or down by 12%. In contrast, the rate of return on the second stock would go up or down by only 3%. Let’s flesh this out a little more by assuming a risk-free rate and market rate of return.

\[\beta = \frac{\sigma_{1M}}{\sigma_M^2} = \frac{\text{Cov}(r, r_M)}{\sigma_M^2}\]

where \(\sigma_{1M}\) is the covariance between the asset and the market.

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\(^4\) To be precise, beta is equal to the covariance of the asset and market returns divided by the variance of the market.
<table>
<thead>
<tr>
<th></th>
<th>Stock 1</th>
<th>Stock 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>risk-free rate</td>
<td>2%</td>
<td>2%</td>
</tr>
<tr>
<td>market rate of return</td>
<td>7%</td>
<td>7%</td>
</tr>
<tr>
<td>beta</td>
<td>2</td>
<td>0.5</td>
</tr>
<tr>
<td>$E(r) = r_f + \beta[r_M - r_f]$ $= 12%$</td>
<td>$4.5%$</td>
<td></td>
</tr>
</tbody>
</table>

Thus, the stock that varies more will have a higher required rate of return (i.e., cost of equity capital) according to CAPM.

Briefly, what would happen if the rate of return on an asset did not vary at all? Thus, when the stock market as a whole moved up or down, this stock’s rate of return would not change. The beta for this stock would be zero. In this case, we would say that the stock’s rate of return carried no risk. Hence, its rate of return should be equal to the risk-free rate of return.

$E(r) = r_f + \beta[r_M - r_f] = r_f + 0[r_M - r_f] = r_f$

On the other hand, suppose there was a stock that moved exactly with the market as a whole. When the market went up by 3%, the rate of return on this stock’s rate of return went up by precisely 3% as well. Or, when the market went down by 8%, the rate of return on this stock went down by exactly 8% as well. If the stock’s rate of return moved exactly with the market, then the beta would be exactly one. Now, if there were such a stock, what should be its rate of return? According to CAPM, the rate of return should be exactly equal to the rate of return on the market for all risky assets (e.g., the stock market as a whole).

$E(r) = r_f + \beta[r_M - r_f] = r_f + 1[r_M - r_f] = r_M$

Thus, when a stock’s return moves more than the market (either up or down), then beta will be greater than one and the holder of the stock will be rewarded (or, penalized) with an expected rate of return greater than the market as a whole. If, on the other hand, a stock’s rate of return tended to move less than the market, then the beta will be less than one and the expected rate of return will be less than that expected on the market as a whole. The underlying idea is that the market rewards one for taking on greater risk – holding stocks that move up and down more than the market as a whole.

In practice, the beta is calculated using the statistical technique known as regression analysis. It is easy enough to perform a regression with Excel. One simply gets historical data on the rate of return for the particular stock and the market as a whole (again, the S&P 500 or some other broad index measure is used) and hits a few keys in
Excel to arrive at the beta. Fortunately for us, betas have become so popular that they are widely publicized. You can find a stock’s beta, for example, on Yahoo!Finance and MSN money central. You will find, however, that any two sources will typically present different measures of beta for the exact same stock. Why? One reason for this is that historical data must be used, which brings up the question of just what historical data to be used (e.g., 1999-2009, 1989-2009, 1929-2009, or what?). A second reason is the index to be used (Dow Jones, S&P 500, Russell 2000, or what?). There are other reasons for the different estimates of beta, but we seem to have enough of an idea already. The point is to know what the beta is telling you and just how to use it.

The CAPM provides a useful way to arrive at the cost of equity capital. The expected rate of return in the CAPM is the appropriate discount rate used to apply to stocks. Thus, if one were using the Gordon Model in order to value a stock, they could find the dividend payment and estimate the growth rate of that dividend, then use the CAPM rate of return as the proper discount rate to apply. For example, suppose we have the following information on a particular stock.

\[
\begin{align*}
D &= \$2 \text{ per share} \\
r_f &= 3\% \\
\beta &= 1.25 \\
g &= 4\% \text{ annually} \\
r_M &= 8\% \\
\end{align*}
\]

According to the Gordon Model, the value of the stock would be the following.

\[
P = \frac{\$2}{r - .04}
\]

Of course, we need the discount rate to use. We get this from the CAPM.

\[
E(r) = r_f + \beta [r_M - r_f] = .03 + 1.25[.08 - .03] = .03 + .0625 = .0925 = 9.25\%
\]

Notice in the above that the risk premium for this stock is 6.25%. Now, we can value the stock.

\[
P = \frac{\$2}{.0925 - .04} = \frac{2}{.0525} = \$38.10
\]

What if the beta had been 0.75 with everything else the same?

\[
E(r) = r_f + \beta [r_M - r_f] = .03 + 0.75[.08 - .03] = .03 + .0375 = .0675 = 6.75\%
\]

The risk premium would drop to only 3.75%, the cost of equity capital falls to 6.75%, and the stock price would rise.

\[
P = \frac{\$2}{.0675 - .04} = \frac{2}{.0275} = \$72.73
\]
Thus, since the rate of return on the stock does not vary as much now (i.e., beta is lower) the value of the stock should increase. In other words, investors should be willing to pay more for a stock when its rate of return does not change much.

We will use the CAPM to arrive at the cost of equity capital for a corporation. The cost of debt capital has already been shown to be simply the yield to maturity (or, interest rate) on a corporation’s debt (e.g., bonds). The two costs then tell us what those people currently holding our debt and equity require in terms of a rate of return. We have deciphered the signals originating in the bond and stock markets to uncover the information we needed in order to evaluate investment projects using either the Net Present Value Method or Internal Rate of Return Method. Now, what do we actually do with these numbers? We want one number to use as our cost of capital. At this point we have two numbers. In the next section we see that we will simply take a weighted average of the two numbers to arrive at the cost of capital (or, now it will make more sense to call it the weighted average cost of capital).

4. Weighted Average Cost of Capital (WACC)

In order to determine a corporation’s cost of capital we will take a simple weighted average of the cost of debt and cost of equity capital. With only a couple of exceptions, there is really not much more to do to arrive at the cost of capital. The current section will demonstrate how to compute the weighted average and make a couple of adjustments to what has been said already.

Anyone that has computed their grade point average (GPA) for a semester knows how to calculate a weighted average. To illustrate, suppose a student received the following grades in a semester.

<table>
<thead>
<tr>
<th>Course</th>
<th>Credit Hours</th>
<th>Letter Grade</th>
<th>Point Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intro to Organizations</td>
<td>3</td>
<td>A</td>
<td>4.0</td>
</tr>
<tr>
<td>Calculus</td>
<td>5</td>
<td>C</td>
<td>2.0</td>
</tr>
<tr>
<td>Field Experience Seminar</td>
<td>2</td>
<td>B</td>
<td>3.0</td>
</tr>
</tbody>
</table>

What is the student’s GPA for this semester? If we just look at the last two columns then we might initially answer 3.0. However, we need to take into account the credit hours for each course, and weigh the grades accordingly. After doing so, we see that the GPA is actually 2.8 rather than 3.0. The calculus course in which the student received a C represented half of the credit hours taken and therefore weighed heavily in determining the GPA.
<table>
<thead>
<tr>
<th>Course</th>
<th>% of Total Credits</th>
<th>Point Grade</th>
<th>GPA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intro to Organizations</td>
<td>30%</td>
<td>4.0</td>
<td>1.2</td>
</tr>
<tr>
<td>Calculus</td>
<td>50%</td>
<td>2.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Field Experience Seminar</td>
<td>20%</td>
<td>3.0</td>
<td>( \frac{6}{2.8} = 2.1 )</td>
</tr>
</tbody>
</table>

Calculating the weighted average cost of capital is no different than calculating your GPA. We simply weight the cost of debt and equity capital according to the percentage that each type of financing represents in the total capital of the corporation. As a very simple example, suppose we have the following for a corporation.

<table>
<thead>
<tr>
<th>Source</th>
<th>Market Value</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Debt (bonds)</td>
<td>$200</td>
<td>5%</td>
</tr>
<tr>
<td>Equity (stocks)</td>
<td>$800</td>
<td>10%</td>
</tr>
</tbody>
</table>

If we took the simple average, we would arrive at a cost of capital of 7.5%. However, it is clear that the cost of equity capital should be weighed more than the cost of debt.

<table>
<thead>
<tr>
<th>Source</th>
<th>% of Total Value</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Debt (bonds)</td>
<td>20%</td>
<td>5%</td>
</tr>
<tr>
<td>Equity (stocks)</td>
<td>80%</td>
<td>10%</td>
</tr>
</tbody>
</table>

The weighted average gets pulled above the simple average because stocks represent a higher percentage in this corporation’s total market value. Notice, we are using the market value of the sources of financing. This will typically differ from what is actually on the balance sheet. For example, the shareholder equity section of the balance reports the amount of equity raised in the initial offering of the stock – hence, the historical value. We are interested in the current market value of the stock. The market value of the stock of a corporation is called its market capitalization and is found by multiplying the current stock price by the number of shares outstanding. The market capitalization is typically provided by Yahoo!Finance and MSN money central. The market value of the debt outstanding can be a bit more difficult to actually find online, but will often be provided in a corporation’s 10K filing with the SEC. The market value of the debt is simply the current value of the bonds outstanding. In applications, this can create an additional step for us if the corporation has more than one type of bond outstanding – but, we’ll see that this will mean calculating another weighted average.
The most significant modification we need to make in order to arrive at the WACC is to adjust for the tax impact of interest. Interest expense is deducted from Earnings before Interest & Taxes (i.e., EBIT) prior to calculating the tax payment. We need to take this into account when using the interest rate as the cost of debt. In order to see the implications, consider the following two firms, identical in all ways except for the capital structure (i.e., their debt and equity). Firm 1 has been all equity financed and carries no debt, and therefore no interest expense. Firm 2 had used some debt to finance its assets. In the current period suppose interest expense had been say $300 – please note, the numbers here are used just to illustrate the tax impact.

<table>
<thead>
<tr>
<th></th>
<th>Firm 1</th>
<th>Firm 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>EBIT</td>
<td>$1,000</td>
<td>$1,000</td>
</tr>
<tr>
<td>- Interest</td>
<td>0</td>
<td>300</td>
</tr>
<tr>
<td>EBT</td>
<td>$1,000</td>
<td>$700</td>
</tr>
<tr>
<td>- Tax</td>
<td>350</td>
<td>245</td>
</tr>
<tr>
<td>= Net Income</td>
<td>$650</td>
<td>$455</td>
</tr>
</tbody>
</table>

Now, what did the interest expense actually cost Firm 2? The net income for Firm 2 was only $195 less than Firm 1, not $300 less. The reason is that the total taxes were lower for Firm 2 even though they both made the same EBIT. The interest expense was deducted prior to calculating the taxes to be paid. In this case, we have used a 35% tax rate for both firms. Now, let’s see how to come up with that $195 difference between the Net Incomes of the two firms. This, after all, is the true cost to Firm 2 for taking on debt and paying interest.

$$\text{Interest} \times (1 - t) = 300 \times (1 - .35) = 195$$

The true cost of interest is found by multiplying interest by one minus the tax rate ($t$). With a tax rate of 35%, Firm 2 ‘loses’ only 65% of the interest expense compared with a situation in which it paid no interest at all. This means that the true cost of debt is not the interest rate, but the interest rate adjusted for the tax benefit.

$$\text{Cost of Debt} = i \times (1 - t)$$

The above is called the after-tax interest rate, or **after-tax cost of debt**. Using a 35% tax rate, we can redo our previous calculation of the WACC.
The WACC has decreased because the true cost of debt is not the interest rate of 5% but rather the after-tax interest rate of 3.25% (assuming a 35% tax rate). A more complete example should be useful at this point.

**Example: B.B. Lean Co.**

The B.B. Lean Co. has 1.4 million shares of common stock outstanding. The stock currently sells for $20 per share. The market value of the firm’s debt is $4.65 million with a current yield to maturity of 11% (i.e., the current interest rate). The risk-free rate is 8%, and overall market rate of return is 15%. The historical CAPM beta for Lean is .74. The corporate tax rate is 34%. What is B.B. Lean Co.’s WACC?

1. Calculate the CAPM required rate of return (or, cost of equity capital).

   \[ r_E = r_f + \beta(r_m - r_f) = 8\% + .74 \times [15\% - 8\%] = 13.18\% \]

2. Calculate the total market value of the equity (or, market capitalization)

   \[(\text{Equity}) \text{ Market Capitalization} = 20 \times 1.4 \text{ million} = \$28 \text{ million} \]

3. Calculate the market value of the debt

   Market Value of Debt = $4.65 million (this has been given to us in the problem)

4. Calculate the total market value of debt and equity

   \[ V = \$28 \text{ million} + \$4.65 \text{ million} = \$32.65 \text{ million} \]

5. Calculate the percentage of equity and debt in the total market value

   Equity Percent = $28/$32.65 = 85.76%

   Debt Percent = $4.65/$32.65 = 14.24%

6. Calculate the WACC (cost of capital)

   \[ WACC = (.8576 \times .1318) + (.1424 \times .11 \times (1 - .34)) = .1234 = 12.34\% \]
Quick Reference

1. Weighted Average Cost of Capital (WACC) --- with Debt and Common Stock

\[ WACC = \frac{D}{V} i(1 - t) + \frac{E}{V} r \]

D is the Market Value of Debt
E is the Market Value of Equity
V is the Total Market Value
i is the yield to maturity (or, cost of debt)
t is the tax rate
r is the cost of equity

2. WACC --- with Debt, Common Stock, Preferred Stock

\[ WACC = \frac{D}{V} i(1 - t) + \frac{C}{V} r_c + \frac{F}{V} r_f \]

D is the Market Value of Debt
C is the Market Value of Common Stock
F is the Market Value of Preferred Stock
V is the Total Market Value
i is the yield to maturity (or, cost of debt)
t is the tax rate
rc is the cost of common equity
rf is the cost of preferred stock

Total Equity Capital = Common Stock + Preferred Stock

Preferred Stock is a hybrid of debt and equity, meaning it has characteristics of both. However, it is listed within the shareholders equity portion of the balance sheet. It is a title of ownership, but normally without any voting rights (thus, no control). It is like debt in the sense that preferred stock normally promises a particular dividend payment each year, unlike common stock where the dividend may go up or down depending upon profitability. Since preferred stock promises a set dividend each year, and it is assumed to last forever the required rate of return (or, cost of this form of equity) is easy to calculate --- it uses our ‘forever’ assumption.

\[ r_f = \frac{D}{P} \]

where D is the annual dividend, and P the current market price.
4. Cost of Debt (or, yield to maturity --- YTM)

\[ P = \frac{C}{(1 + YTM)^1} + \frac{C}{(1 + YTM)^2} + \cdots + \frac{C}{(1 + YTM)^T} + \frac{FV}{(1 + YTM)^T} \]

\( P \) is the current market price of the bond  
\( C \) is the annual coupon payment  
\( FV \) is the face value of the bond  
\( T \) is the year the bond matures

5. Cost of Equity \((r\), using Gordon Model\)

\[ r = \frac{D}{P} + g \]

\( D \) is the annual dividend payment  
\( P \) is the current market price  
\( g \) is the expected annual growth rate of the dividend

6. Cost of Equity \((r\), using CAPM\)

\[ r = r_f + \beta(r_M - r_f) \]

\( r_f \) is the risk-free rate (e.g., interest rate on government bond)  
\( r_M \) is the rate of return on the market for risky assets as a whole (e.g., S&P 500)  
\( \beta \) is beta (see text for explanation)

7. Net Present Value (NPV)

\[ NPV = \left[ \frac{CF_1}{(1 + \text{WACC})^1} + \frac{CF_2}{(1 + \text{WACC})^2} + \cdots + \frac{CF_T}{(1 + \text{WACC})^T} \right] - \text{Cost} \]

\( \text{WACC} \) is the cost of capital (weighted average cost of capital)  
\( CF \) is the expected cash flow for a particular year  
\( \text{Cost} \) is the initial cost of the investment project  
\( T \) the economic life of the investment project

8. Internal Rate of Return (IRR)

\[ \text{Cost} = \left[ \frac{CF_1}{(1 + IRR)^1} + \frac{CF_2}{(1 + IRR)^2} + \cdots + \frac{CF_T}{(1 + IRR)^T} \right] \]